

ENTRANCE EXAMINATION, FEBRUARY 2014  
QUESTION PAPER BOOKLET

Ph.D. (Physics)

Marks: 75

Time: 2.00 hrs.

Hall Ticket No.:

I. Please enter your Hall Ticket Number on Page 1 of this question paper and on the OMR sheet without fail.

II. Read carefully the following instructions:

1. This Question paper has two Sections: **Section A** and **Section B**.
2. **Section A** consists of 25 objective type questions of one mark each.  
**There is negative marking of 0.33 mark for every wrong answer.**  
The marks obtained by the candidate in this Section will be used for resolving the tie cases.
3. **Section B** consists of 50 objective type questions of one mark each.  
**There is no negative marking in this Section.**
4. Answers are to be marked on the OMR answer sheet following the instructions provided there upon.
5. Only Scientific Calculators are permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
6. Hand over the OMR answer sheet at the end of the examination to the Invigilator.

This book contains 24 pages

III. Values of physical constants:

$$c = 3 \times 10^8 \text{ m/s}; h = 6.63 \times 10^{-34} \text{ J.s}; k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$e = 1.6 \times 10^{-19} \text{ C}; \mu_0 = 4\pi \times 10^{-7} \text{ Henry/m}; \epsilon_0 = 8.85 \times 10^{-12} \text{ Farad/m}$$

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## SECTION - A

- The value of the line integral of the vector  $\vec{A} = (2x\hat{i} + 3x\hat{j} - z\hat{k})$  from the point  $P(5, 0, 0)$  to  $Q(0, 5, 0)$  along the straight line path  $PQ$  is
  - 12.5.
  - 25.
  - 0.
  - 50.
- The equation  $\left| \frac{z-3}{z+3} \right| = 2$  (where  $z$  is a complex variable) represents
  - a straight line passing through the point  $(-3, 3)$ .
  - a pair of straight lines.
  - a circle of radius 4.
  - an ellipse with a major axis of length 3 and minor axis of length 2.

- The singularities of the function

$$\frac{\ln(z-2)}{(z^2-2z+2)^2}$$

are:

- Second-order poles at  $z = 1 \pm i$  and a branch point singularity at  $z = 2$
  - Second-order poles at  $z = -1 \pm i$  and a branch point singularity at  $z = 2$
  - Second-order poles at  $z = -1 \pm i$  and branch point singularity at  $z = -2$
  - Second-order poles at  $z = 1 \pm i$  and a branch point singularity at  $z = -2$
- The value of the integral  $\int_0^{\infty} x^3 e^{-x} dx$  is given by
    - 3.
    - 6.
    - $\sqrt{\pi}$ .
    - $\sqrt{\pi}/3$ .

5. Consider a classical particle of mass  $m$  moving in a central field of force which can be derived from a potential  $V(r)$ . If we consider the radial motion only, then what is the effective potential in which the radial motion with angular momentum  $l$  occurs?

A.  $V(r) + \frac{l}{2mr}$

B.  $V(r) + \frac{l^2}{2mr}$

C.  $V(r) + \frac{2l^2}{mr^2}$

D.  $V(r) + \frac{l^2}{2mr^2}$

6. A rigid body consists of 8 point masses sitting at the vertices of a regular octagon. How many degrees of freedom does the system have in a three-dimensional space?

A. 24

B. 16

C. 11

D. 6

7. The Lagrangian of a system is given by:

$$L = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + k(x_1^2x_2 - x_1x_2^2).$$

If the canonical momenta corresponding to  $x_1$  and  $x_2$  are  $p_1$  and  $p_2$  respectively, then the Hamiltonian corresponding to  $L$  is given by

A.  $\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + k(x_1^2x_2 - x_1x_2^2).$

B.  $\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + k(x_1^2x_2 + x_1x_2^2).$

C.  $\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - kx_1^2x_2 + kx_1x_2^2.$

D.  $\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - kx_1^2x_2 - kx_1x_2^2.$

8. It is given that the potential at a point  $\vec{r}$  due to a charge distribution is

$$\varphi = \frac{qe^{-kr}}{r}$$

The electric field  $\vec{E}(\vec{r})$  is then given by

- A.  $\frac{q}{r^2}e^{-kr} \left( \frac{1}{r} + k \right) \vec{r}$ .
- B.  $\frac{q}{r^2}e^{-kr} \left( \frac{2}{r} + k \right) \vec{r}$ .
- C.  $\frac{q}{r}e^{-kr} \left( r + \frac{k}{r} \right) \vec{r}$ .
- D.  $\frac{q}{r}e^{-kr} \left( 1 + \frac{k}{r^2} \right) \vec{r}$ .
9. The ratio of skin depth in copper at 1000 Hz to that at  $10^8$  Hz is approximately equal to
- A.  $3.1 \times 10^4$ .
- B.  $2.1 \times 10^2$ .
- C.  $3.1 \times 10^2$ .
- D.  $10^5$ .
10. The vector potential in a certain region of space is given by

$$\vec{A}(\vec{r}, t) = K(y \cos \omega t \hat{i} + x \sin \omega t \hat{j}).$$

The magnetic field  $\vec{B}(\vec{r}, t)$  is then given by

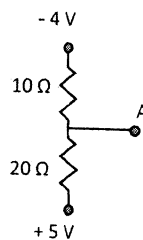
- A.  $K[\sin(\omega t) + \cos(\omega t)]\hat{k}$ .
- B.  $K[\sin(\omega t) - \cos(\omega t)]\hat{k}$ .
- C.  $K[\sin(\omega t) - \cos(\omega t)]\hat{j}$ .
- D.  $K[\sin(\omega t) + \cos(\omega t)]\hat{i}$ .
11. What is the magnitude of the magnetic induction  $B$  at the centre of a current loop of radius  $a$  carrying a current  $I$ ?
- A.  $\frac{\mu_0 I}{2a}$
- B.  $\frac{\mu_0 I}{2a^2}$
- C.  $\frac{\mu_0 a}{2I}$
- D.  $\frac{\mu_0 a^2}{2I}$

12. For the wave function  $\psi = Ae^{-ax^2 - i\frac{E}{\hbar}t}$ , the probability current density  $\vec{J}$  is such that
- $\vec{J} = 0$ .
  - $\vec{J} \neq 0$  but  $\vec{\nabla} \cdot \vec{J} = 0$ .
  - $\vec{\nabla} \cdot \vec{J} \neq 0$  but  $\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$ .
  - $\vec{J}$  is non-zero but constant.
13. The total probability  $P = \int d^3x \psi^*(\vec{x}, t)\psi(\vec{x}, t)$  is conserved
- for all states and under unitary time evolution.
  - only for stationary states and under unitary time evolution.
  - for all states and under both unitary and non-unitary time evolution.
  - only for stationary states and under both unitary and non-unitary time evolution.
14. For what value of  $\epsilon$  does  $e^{i\epsilon kx - i\omega t}$  (where  $k > 0$ ,  $\omega > 0$ ) represent a plane wave traveling along the negative  $x$ -axis?
- $\epsilon = 1$
  - $\epsilon = -1$
  - $\epsilon = i$
  - $\epsilon = -i$
15. If  $\psi_0$  is the ground state wave function of the Hamiltonian  $H$  belonging to the eigenvalue  $E_0$  and  $\phi$  is any arbitrary normalized wave function (different from  $\psi_0$ ) then  $\langle \phi | H | \phi \rangle$
- is always lower than  $E_0$ .
  - can never be higher than  $E_0$ .
  - may be higher or lower than  $E_0$  depending on  $\phi$ .
  - is always higher than  $E_0$ .
16. The specific heat of a non-magnetic metal can be expected to behave at low temperature as
- $AT + BT^3$ .
  - $AT + Be^{-k_B T/E}$ .
  - $AT + BT^2$ .
  - $AT^2 + BT^3$ .

17. The paramagnetic susceptibility of a free electron gas at low temperature is
- A. proportional to temperature  $T$ .
  - B. proportional to  $\frac{1}{T}$ .
  - C. proportional to  $T^2$ .
  - D. independent of  $T$ .
18. Consider a system of a particle in a two-level system with energy levels as 0 and  $E$ . The average energy of the system is
- A.  $\frac{E}{2}$ .
  - B.  $\frac{E}{2}k_B T$ .
  - C.  $\frac{Ee^{-E/k_B T}}{1 + e^{-E/k_B T}}$ .
  - D.  $E(1 + e^{-E/k_B T})$ .
19. How many branches of elastic (phonon) modes of vibration are possible along the [111] direction of a Germanium crystal that crystallizes in diamond structure?
- A. 3 acoustic branches only
  - B. 3 optical branches only
  - C. 3 acoustic branches and 3 optical branches
  - D. 3 acoustic branches and 6 optical branches
20. The harmonic approximation for thermal vibrations of atoms in a crystal lattice cannot explain the following property of solids:
- A. Thermal expansion
  - B. Electrical conductivity
  - C. Dulong and Petit law
  - D. Debye specific heat

21. A spectrum for a molecule was recorded in the infrared region. This has to do with the
- transition between two electronic levels.
  - transition between the vibrational levels of one electronic state to the vibrational state of another electronic level.
  - transition between the vibrational levels of one electronic state to another vibrational state of the same electronic level.
  - transition between the rotational levels of one vibrational state to a rotational state of the same vibrational state.
22. The normal and anomalous Zeeman effects refer to transitions between
- two electronic levels and their vibrational levels respectively in the presence of an external electric field.
  - atomic states where only singlet and non-singlet states respectively are involved in the presence of an external magnetic field.
  - two electronic levels and their vibrational levels respectively in the presence of an external magnetic field.
  - transitions between vibrational and rotational levels respectively in the presence of an external magnetic field.
23. Among the following, which particles do *not* undergo strong interaction?
- Baryons
  - Mesons
  - Hadrons
  - Leptons
24. In the figure below, what is the voltage at the point marked A? (All voltages are referenced to ground):

- +0.5 V
- +2.0 V
- 1.0 V
- 0.5 V



25. When the voltage gain of an amplifier is decreased, the band-width
- increases.
  - decreases.
  - is not affected.
  - becomes discrete.



## SECTION - B

26. The coefficient of  $(x - 2)^3$  in the Taylor expansion of  $e^x$  around  $x = 2$  is

- A.  $\frac{1}{3!}$ .
- B.  $\frac{2^3}{3!}$ .
- C.  $\frac{e^{-2}}{3!}$ .
- D.  $\frac{e^2}{3!}$ .

27. The differential equation

$$x \frac{dy}{dx} + 6y = 0$$

under the change of variable from  $x$  to  $t$  with  $x = t^2$  becomes

- A.  $t^2 \frac{dy}{dt} + 6y = 0$ .
- B.  $t \frac{dy}{dt} + 12y = 0$ .
- C.  $\frac{dy}{dt} + 6t^2 y = 0$ .
- D.  $t \frac{dy}{dt} + 3y = 0$ .

28. The operator defined as

$$\nabla = \frac{\partial}{\partial x} + i \frac{\partial}{\partial y}$$

is equal to

- A.  $\frac{\partial}{\partial z}$ .
- B.  $\frac{\partial}{\partial z} + \frac{\partial}{\partial z^*}$ .
- C.  $2 \frac{\partial}{\partial z^*}$ .
- D.  $2 \frac{\partial}{\partial z}$ .

29. What is the residue of

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$$

at  $z = 2i$ ?

- A.  $\frac{-4 - 4i}{(2i + 1)^2 i}$
- B.  $\frac{7 + i}{25}$
- C.  $\frac{7 - 2i}{25}$
- D.  $\frac{-4 - 4i}{(2i + 1)^2}$

30. The value of  $\nabla^2 \left[ \vec{\nabla} \cdot \left( \frac{\vec{r}}{r^2} \right) \right]$  is equal to

- A.  $\frac{1}{r^4}$ .
- B.  $\frac{4}{r^4}$ .
- C.  $\frac{3}{r^4}$ .
- D.  $\frac{2}{r^4}$ .

31. The value of the integral  $\oint \frac{e^z}{(z^2 + \pi^2)^2} dz$  over a closed contour which is a circle  $|z| = 4$  is:

- A.  $i/\pi$ .
- B.  $i\pi$ .
- C.  $2\pi^2 i$ .
- D.  $\pi^2 i$ .

32. The Jacobian of the transformations:

$$x = \frac{u+v}{\sqrt{2}}, \quad y = \frac{u-v}{\sqrt{2}}$$

is equal to

- A. 1.
- B. 0.
- C.  $\sqrt{2}$ .
- D. 2.

33. Which of the following equations:

(i) Newton's equations ; (ii) Maxwell's equations

is/are invariant under the transformations:  $\vec{x}' = \vec{x} - \vec{v}t$ ,  $t' = t$ ?

- A. Both (i) and (ii).
- B. Neither (i) nor (ii).
- C. Only (i).
- D. Only (ii).

34. A particle of mass  $m$  moves in a circular orbit of radius  $a$  under the action of a central force whose potential is  $V(r) = b^2mr^4$ , where  $b$  is a positive constant. For what angular momentum will the orbit be a circle of radius  $a$ ?

- A.  $4bma^3$ .
- B.  $2ma^2\sqrt{b}$ .
- C.  $2mb^2a^3$ .
- D.  $2bma^3$ .

35. The Lagrangian

$$L = e^{\gamma t} \left[ \frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2 \right]$$

describes a

- A. simple harmonic oscillator of amplitude  $\gamma$ .
- B. damped-driven oscillator of damping factor  $\gamma$  and driving force  $\sin \gamma t^2$ .
- C. driven oscillator of driving force  $\sin \gamma t$ .
- D. damped harmonic oscillator of damping factor  $\gamma$ .

36. A sphere of mass  $m$  moves in a tube that rotates in the  $x - y$  plane about the  $z$ -axis with a constant angular velocity  $\omega$ . The Lagrange equation for the system is given by

- A.  $\ddot{r} + \omega^2r = 0$ .
- B.  $\ddot{r} - \omega^2r = 0$ .
- C.  $\ddot{r} + \omega^2r \sin \omega t = 0$ .
- D.  $\ddot{r} - \omega^2r \cos \omega t = 0$ .

37. For what value of  $\alpha$ , the transformations:  $q = \sqrt{(2\alpha P)} \sin Q$ ,  $p = \sqrt{(2\alpha P)} \cos Q$ , would be canonical?

- A. 1
- B. 1/2
- C. 2
- D. 0

38. The Hamiltonian for a three-dimensional isotropic harmonic oscillator is given by

$$\frac{1}{2} [p_1^2 + p_2^2 + p_3^2 + \mu^2(q_1^2 + q_2^2 + q_3^2)] .$$

$G = \mu q_1 \cos(\mu t) + \epsilon p_1 \sin(\mu t)$ , is a constant of motion when

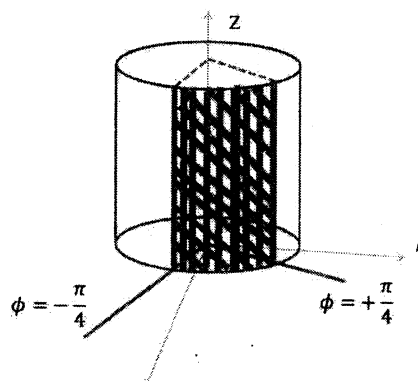
- A.  $\epsilon = 0$ .
- B.  $\epsilon = 1$ .
- C.  $\epsilon = -1$ .
- D.  $\epsilon = \frac{1}{2}$ .

39. Assume that our space contains a positive unit charge distributed uniformly over the surface of a sphere of radius  $a$ . The corresponding electrostatic energy in Gaussian units is given by

- A.  $1/a$ .
- B.  $1/\pi a^2$ .
- C.  $1/2a$ .
- D.  $1/4\pi a$ .

40. A radial field  $\vec{B} = \frac{3}{\rho} \cos \phi \hat{\rho}$  exists in free space. What is the magnetic flux  $\Phi$  (in Wb) crossing a cylindrical surface (shown in figure) defined by  $-\pi/4 \leq \phi \leq \pi/4$ ,  $0 \leq z \leq 1m$ ?

- A. 0.0
- B. 4.24
- C. 42.4
- D. 424.0



41. Consider two concentric spherical shells of inner radius 0.1 m and outer radius 1.0 m. The inner shell is maintained at potential 1 V while the outer shell is grounded. The potential in the free space between the two shells is given (in Volt) by

A.  $-\frac{1}{9} \left( \frac{1}{r} - 1 \right)$ .

B.  $-\frac{1}{9} \left( \frac{1}{r} + 1 \right)$ .

C.  $\frac{1}{9} \left( \frac{1}{r} + 1 \right)$ .

D.  $\frac{1}{9} \left( \frac{1}{r} - 1 \right)$ .

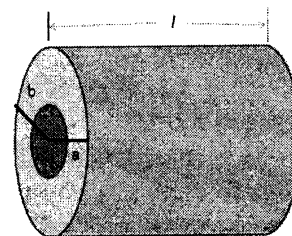
42. What is the resistance of insulation (of conductivity  $\sigma$ ) in a length  $l$  of a coaxial cable as shown in the figure below?

A.  $\frac{1}{2\pi\sigma} \ln \left( \frac{b}{a} \right)$

B.  $\frac{1}{2\pi l\sigma} \ln \left( \frac{b}{a} \right)$

C.  $\frac{1}{l\sigma} \ln \left( \frac{2\pi b}{\sigma a} \right)$

D.  $\frac{2\pi}{l\sigma} \ln(ab/l^2)$



43. It is given that the electric field in  $\vec{E} = E_0 \sin(\omega t - \beta z) \hat{j}$  in free space. What is the magnetic field  $\vec{H}$  at  $t = 0$ ?

A.  $\frac{\beta E_0}{\omega \mu_0} \sin(\beta z) \hat{i}$ .

B.  $\frac{\beta E_0}{\omega \mu_0} \sin(\beta x) \hat{k}$ .

C.  $\frac{\beta E_0}{\omega \mu_0} \cos(\beta z) \hat{i}$ .

D.  $\frac{\beta E_0}{\omega \mu_0} \cos(\beta x) \hat{k}$ .

44. It is given that the electric field is  $\vec{E}(z, t) = 50 \cos(\omega t - \beta z) \hat{i}$  (V/m) in free space. What is the average power crossing a circular area of radius 2.5 m in the plane  $z = \text{constant}$ ? (Given that  $\sqrt{\mu_0/\epsilon_0} = 120\pi \Omega$ .)

A. 56.2 W.

B. 65.2 W.

C. 12.6 W.

D. 24.6 W.

45. If  $a$  and  $a^\dagger$  are the lowering and raising operators respectively in the case of a harmonic oscillator, and  $U = e^S$ , where  $S = \alpha a^\dagger - \alpha^* a$ ,  $\alpha$  being a constant, then  $U^\dagger a U$  is given by
- $a - \alpha$ .
  - $a^\dagger + \alpha$ .
  - $a - \alpha^* \alpha$ .
  - $a + \alpha$ .
46. A spin  $\frac{1}{2}$  particle is at rest with spin  $s_z$  pointing "up" (i.e.  $s_z = +\frac{1}{2}$ ). After a rotation of the coordinates by an angle  $\theta$  about  $y$ -axis, the probability that  $s_z$  will be pointing "down" (i.e.,  $s_z = -\frac{1}{2}$ ) is
- $\cos^2 \frac{\theta}{2}$ .
  - $\sin^2 \frac{\theta}{2}$ .
  - $\sin^2 \theta$ .
  - $\sin \frac{\theta}{2} \cos \frac{\theta}{2}$ .
47. Consider the gravitational force between the proton and the electron in the hydrogen atom as a perturbation. It is given that the mass of the electron is  $m$ , the mass of the proton is  $M$  and the ground state wave function of the hydrogen atom is  $\psi = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$ , where  $a$  is a constant. What is the first-order perturbative correction to the ground state energy?
- $\frac{-GMm}{2a}$
  - $\frac{-2GMm}{a}$
  - $\frac{-GMm}{a}$
  - $\frac{GMm}{a}$
48. A particle is acted on by a force  $F = -kx$  along the  $x$ -axis. The energy ( $E_n$ ) for this particle in  $WKB$  approximation depends on the quantum number  $n$  as:  $E_n \propto n^x$ , where  $x$  is equal to
- $-\frac{1}{2}$ .
  - 2.
  - +1.
  - $+\frac{1}{2}$ .

49. Which of the operators,  $\hat{A}_1$  and  $\hat{A}_2$  defined by

$$\hat{A}_1\phi(x) \equiv \phi^2(x) \quad ; \quad \hat{A}_2\phi(x) \equiv \sin[\phi(x)]$$

is/are linear?

- A.  $\hat{A}_1$
- B.  $\hat{A}_2$
- C. Both  $\hat{A}_1$  and  $\hat{A}_2$
- D. Neither  $\hat{A}_1$  nor  $\hat{A}_2$

50. Two identical spin-0 particles are in an infinite square well potential of length  $a$ . One of them is in the ground state and the other in the first excited state. The wave function  $\psi$  for the system is given by

- A.  $\psi(x_1, x_2) = \sin\left(\frac{\pi x_1}{a}\right) + \sin\left(\frac{2\pi x_2}{a}\right)$ .
- B.  $\psi(x_1, x_2) = \sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right)$ .
- C.  $\psi(x_1, x_2) = \sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right) + x_1 \leftrightarrow x_2$ .
- D.  $\psi(x_1, x_2) = \sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right) - x_1 \leftrightarrow x_2$ .

51. The state of a one-dimensional harmonic oscillator at time  $t = 0$  is given by

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle),$$

where  $|n\rangle$  is the  $n$ -th energy eigenstate. Then the expectation value of its energy at time  $t$  is

- A.  $\frac{1}{2\sqrt{3}}\hbar\omega$ .
- B.  $\frac{3}{2}\hbar\omega$ .
- C.  $3\hbar\omega$ .
- D.  $\frac{3}{2}\hbar\omega e^{-2i\omega t}$ .

52. A particle of mass  $m$  is bound in one dimension by the potential  $F|x|$ , where  $F$  is a positive constant. What would be the variational energy of the ground state of the system if you choose a trial function as

$$\psi = \left(\frac{2\alpha^2}{\pi}\right)^{1/4} e^{-\alpha^2 x^2/2} ?$$

- A.  $\left(\frac{\hbar^2 F^2}{2m}\right)^{1/3}$   
 B.  $\left(\frac{\hbar^2 F^2}{2\pi m}\right)^{1/3}$   
 C.  $2\left(\frac{\hbar^2 F^2}{\pi m}\right)^{1/3}$   
 D.  $1.5\left(\frac{\hbar^2 F^2}{2\pi m}\right)^{1/3}$
53. A classical system of  $N$  distinguishable non-interacting particles each of mass  $m$  is placed in a three-dimensional harmonic well:  $V(r) = r^2/2b$ . The canonical partition function per particle of the system is proportional to
- A.  $T$ .  
 B.  $T^2$ .  
 C.  $T^3$ .  
 D.  $T^4$ .
54. Consider an ideal gas of  $N$  identical particles held at temperature  $T$ . If  $E_k$  denotes the energy of a particle in state  $k$ , the Helmholtz free energy of the system is given by

- A.  $-Nk_B T \ln \left( \sum_k e^{-E_k/k_B T} \right)$ .  
 B.  $-Nk_B T \ln \left( \sum_k e^{-E_k/k_B T} \right) + k_B T \ln N$ .  
 C.  $-Nk_B T \ln \left( \sum_k e^{-E_k/k_B T} \right) + k_B T \ln N!$ .  
 D.  $-Nk_B T \ln \left( \sum_k e^{-E_k/k_B T} \right) + Nk_B T \ln N!$ .



55. An ideal gas of  $N$  particles, each of mass  $m$ , at temperature  $T$  is subjected to an external force whose potential energy has the form  $V = \Lambda x$ , with  $\Lambda > 0$ , and  $0 \leq x \leq \infty$ . What is the average potential energy per particle?
- A.  $\frac{1}{2} k_B T$
  - B.  $k_B T$
  - C.  $\frac{3}{2} k_B T$
  - D.  $3k_B T$
56. A harmonic oscillator with energy levels  $E_n = (n + \frac{1}{2}) \hbar\omega$  is in thermal contact with a heat bath at temperature  $T$ . What is the ratio of the probability of the oscillator being in the first excited state to the probability of its being in the ground state?
- A.  $e^{-\hbar\omega/k_B T}$
  - B.  $e^{-2\hbar\omega/k_B T}$
  - C.  $\frac{1}{2}$
  - D.  $e^{-\hbar\omega/2k_B T}$
57. Consider a system of  $N$  independent particles, each of spin  $\frac{1}{2}$  and magnetic moment  $\mu$ , located in a magnetic field  $B$ . The average energy of the system is given by
- A.  $-N\mu B \tanh(\mu B/k_B T)$ .
  - B.  $-N\mu B \tanh(\mu B/2k_B T)$ .
  - C.  $N\mu B \tanh(\mu B/2k_B T)$ .
  - D.  $-N\mu B \coth(\mu B/k_B T)$ .
58. Which of the following lines does the diffraction pattern of metallic sodium likely to contain?
- A. (111)
  - B. (110)
  - C. (220)
  - D. (200)

59. It is given that the Fermi energy of Na is 3.1 eV and the average relaxation time for electron's collision in Na is  $3.3 \times 10^{-14}$  sec. What is the conductivity of Na in e.s.u.?
- A.  $2.1 \times 10^{12} \text{ sec}^{-1}$   
 B.  $2.1 \times 10^{17} \text{ sec}^{-1}$   
 C.  $2.1 \times 10^{19} \text{ sec}^{-1}$   
 D.  $2.1 \times 10^{14} \text{ sec}^{-1}$
60. A beam of electrons with kinetic energy 1 keV is diffracted as it passes through a metal foil of cubic crystal structure with a spacing 1 Å. What is the approximate value of the Bragg angle for the first-order diffraction maximum?
- A.  $5.7^\circ$   
 B.  $30^\circ$   
 C.  $17.5^\circ$   
 D.  $11.2^\circ$
61. Consider a one-dimensional metal of length  $L$  with one free electron per atom and a lattice spacing  $d$  at  $T = 0$  K. The Fermi energy of this system is proportional to
- A.  $d$ .  
 B.  $d^{-1}$ .  
 C.  $d^{-2}$ .  
 D.  $d^{-1/2}$ .
62. According to London's equation, for a superconductor:

$$\frac{\partial \vec{j}}{\partial t} = \left( \frac{c^2}{4\pi\lambda_L^2} \right) \vec{E}.$$

Then it follows that

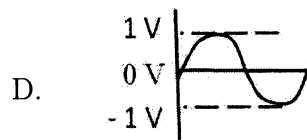
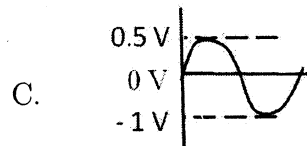
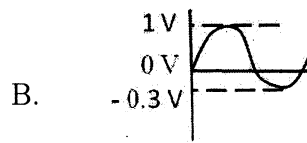
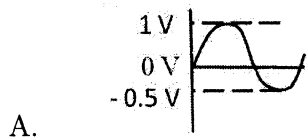
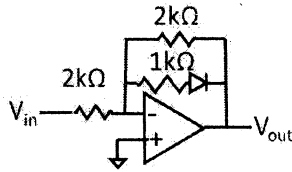
- A.  $\lambda_L = \left( \frac{4\pi mc^2}{nq^2} \right)^{1/2}$   
 B.  $\lambda_L = \left( \frac{nc^2}{4\pi mq^2} \right)^{1/2}$   
 C.  $\lambda_L = \left( \frac{mc^2}{4\pi nq^2} \right)^{1/2}$   
 D.  $\lambda_L = (mc^2 4\pi nq)^{1/4}$

63. In a certain reference frame, a particle has a total energy of 5 GeV and a momentum of 3 GeV/c ( $c$  = velocity of light). What are the rest mass of the particle and its velocity respectively?
- A.  $\frac{4\text{GeV}}{c^2}, \frac{3}{5}c$   
 B.  $\frac{4\text{GeV}}{c^2}, \frac{2}{5}c$   
 C.  $\frac{9\text{GeV}}{c^2}, \frac{1}{5}c$   
 D.  $\frac{4\text{GeV}}{c^2}, \frac{2}{5}c$
64. For two isospin  $\frac{1}{2}$  particles,  $\vec{I}^{(1)} \cdot \vec{I}^{(2)}$  in triplet and singlet states respectively are
- A.  $\frac{3}{4}, -\frac{1}{4}$   
 B.  $\frac{1}{4}, -\frac{3}{4}$   
 C.  $-\frac{3}{4}, \frac{1}{4}$   
 D.  $\frac{1}{4}, \frac{3}{4}$
65. A 2 MeV neutron is emitted in a fission reactor. If it loses half its kinetic energy in each collision with a moderator atom, how many collisions must it undergo to reduce its energy to 0.039 eV?
- A. 10  
 B. 55  
 C. 40  
 D. 26
66. The following reaction
- $$K^- + p \longrightarrow n + \Lambda^0$$
- is forbidden. The reason is the non-conservation of
- A. strangeness.  
 B. isospin.  
 C. baryon number.  
 D. energy.

67. An atom is capable of existing in two states: a ground state of mass  $M$  and an excited state of mass  $M + \Delta$ . If the transition from the ground to the excited state proceeds by the absorption of a photon, the photon frequency  $\nu$  in the laboratory, where the atom is initially at rest is
- A.  $\nu = \frac{c^2 \Delta}{h} \left( 1 + \frac{\Delta}{2M} \right)$ .
- B.  $\nu = \frac{c \Delta}{h} \left( 1 + \frac{\Delta}{2M} \right)$ .
- C.  $\nu = \frac{c^2 \Delta}{h} \left( 1 - \frac{\Delta}{2M} \right)$ .
- D.  $\nu = \frac{c^2 \Delta^2}{2Mh}$ .
68. The full Doppler width of the emission line from argon atoms ( $A = 40, Z = 18$ ) (of wavelength  $\lambda = 500$  nm) at temperature  $T = 300$  K is approximately
- A. 0.144 nm.
- B. 1.44 nm.
- C. 14.4 nm.
- D. 0.0144 nm.
69. An atomic level emitting photons is found to have a lifetime of 1 microsecond. The minimum linewidth of the emission is expected to be around
- A.  $2 \times 10^{10}$  Hz.
- B.  $2 \times 10^9$  Hz.
- C.  $2 \times 10^7$  Hz.
- D.  $2 \times 10^5$  Hz.
70. Three methods are available for numerically solving a polynomial  $f(x) = 0$ , with the following convergence relations respectively:
- (1)  $|\epsilon_{n+1}| = M_1 \epsilon_n^2$ ,
- (2)  $|\epsilon_{n+1}| = M_2 \epsilon_n$ , and
- (3)  $|\epsilon_{n+1}| = M_3 \epsilon_n^\alpha$ ,
- where  $\alpha = 1.618$  and  $M_1, M_2, M_3$  are constants. The order in which they converge, from slowest to fastest, is
- A. 1,2,3.
- B. 2,3,1.
- C. 1,3,2.
- D. 3,2,1.

71. The load line of a transistor circuit is given by  $2V_{CE} + I_C = 20$ , where  $V_{CE}$  and  $I_C$  are measured in  $V$  and  $mA$  respectively. What are the values of  $V_{CC}$  and  $R_C$  for this circuit if  $R_E = 0$ ?
- A. 10 V, 500  $\Omega$
  - B. 20 V, 2 k $\Omega$
  - C. 12 V, 4.7 k $\Omega$
  - D. 10 V, 400  $\Omega$
72. The minimum number of logic gates that are required to implement the following logic function  $F(A, B, C) = \Sigma(0, 2, 4)$  with don't care condition  $D(A, B, C) = \Sigma(3, 6)$  is
- A. 5.
  - B. 3.
  - C. 2.
  - D. 1.
73. The small signal parameter  $h_{fe}$  of a bipolar transistor is 80. When the base current is increased by about 16  $\mu A$ , the collector current increases by
- A. 15 mA.
  - B. 1.3 mA.
  - C. 15  $\mu A$ .
  - D. 0.2  $\mu A$ .
74. An Operational Amplifier (Op-Amp) with open loop gain of  $A_{OL} = 100,000$ , input impedance of  $Z_{in[OL]} = 1 M\Omega$  and output impedance of  $Z_{out[OL]} = 50\Omega$  is used to construct a voltage follower. What are the effective input impedance ( $Z_{in[VF]}$ ) and output impedance ( $Z_{out[VF]}$ ) of this voltage follower?
- A. 1 M $\Omega$  and 50  $\Omega$
  - B. 100 G $\Omega$  and 500  $\mu\Omega$
  - C. 100 k $\Omega$  and 500  $\Omega$
  - D. Infinity and Zero

75. A sinusoidal signal of amplitude 1 V is input to the following operational amplifier circuit, where the diode is an ideal one. What is the output?



FOR ROUGH WORK ONLY

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