Entrance Examination : M.Sc. Statistics, 2014

Hall Ticket Number [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

Time : 2 hours
Max. Marks. 100

Part A : 25 marks
Part B : 75 marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR answer sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet after the examination.
5. There are plain sheets in the booklet for rough work,no additional sheets will be provided.
6. There are a total of 50 questions in Part A and Part B together.
7. **Each question in Part - A has only one correct option and there is negative marking.**
8. **In Part - B, some questions have <u>more than</u> one correct option. All the correct options have to be marked in the OMR answer sheet, otherwise zero marks will be credited.**
9. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
10. The question papers can be taken by the candidates at the end of the examination.

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Part-A

- **Find the correct answer and mark it on the OMR sheet. Each correct answer gets 1 (one) mark and wrong answer gets −0.33 marks.**

1. Read the following statements about Statistics: It is a field of study concerned with

   (1) collection of data.

   (2) computing and making pictorial displays of data.

   (3) making inferences from the data using probability theory depending on how the data are obtained.

   Which of these descriptions fit Statistics most?

   (A) Only (1).

   (B) (1) and (2) , but not (3)

   (C) Only (3).

   (D) All three (1),(2) and (3).

2. Of the 100 students in a class 65 know Telugu , 35 know Hindi , 27 of them know both the languages,the number of students who do not any of the two languages

   (A) is 0 (B) is 23

   (C) is 27 (D) can not be determined from the data given.

3. The number of different arrangements of N distinct objects in a circle is

   (A) $N!$. (B)$(N-1)!$.

   (C) N^N. (D) $(N-1)^{N-1}$.

4. If every arrangement of seating of 4 girls and 4 boys is equally likely,then the probability of getting an arrangement in which no 2 boys are next to each other

   (A) is more than 3/4 (B) is more than 1/30 but less than 1/20

   (C) is less than 1/30 (D) can not be determined from the data given.

5. Two boxes contain 10 balls which are marked $1, 2, \ldots, 10$. One ball is picked up from each box, what is the probability that the difference between the numbers on the two balls is 1 is in the interval

   (A) $(0, 1/10]$. (B) $(1/10, 2/10)$.

   (C) $[2/10, 2/3)$. (D) $(2/3, 1)$.

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6. The mean marks and standard deviation (st.dv.) of marks of two groups of students are as follows:

Group	1	2
mean	47	55
st.dv.	13	27

, one can say that

(A) Group 2 is more competent than Group 1 because their mean marks are more.

(B) Some people in Group 2 might have got marks much higher than the best in Group 1.

(C) Group 1 is more competent than Group 2.

(D) Group 1 and Group 2 are equally competent.

7. The mean of n numbers $a_1, a_2, \ldots, a_n$ is **m** and their standard deviation is s , how should the numbers be transformed to $b_1, b_2, \ldots, b_n$ so that their mean is **4m** and their standard deviation is **2s**?

(A) $b_i = 2a_i$, $i = 1, 2, \ldots, n$
(B) $b_i = 4a_i$, $i = 1, 2, \ldots, n$
(C) $b_i = 2a_i + \mathbf{m}$, $i = 1, 2, \ldots, n$
(D) $b_i = 2a_i + 2\mathbf{m}$, $i = 1, 2, \ldots, n$

8. The median of 9 distinct positive numbers was reported to be 42 , and the next number is 57 upon looking at the numbers again , it was realized that the number 72 was by mistake recorded as 27 , after making this correction , the median will be

(A) 57.
(B) 27.
(C) 72.
(D) 42.

9. Less than half of the students in a class got 50% or more marks in an exam , so the median marks

(A) are greater than 50% but less than 60%.
(B) are greater than 60%.
(C) are less than 50%.
(D) could be 50%.

10. In any numeric data set,

(A) mode need not be unique whereas mean and median are.

(B) neither mean nor median nor mode need be unique.

(C) mean need not be unique, but median and mode are.

(D) all the three measures, mean, median, and mode are unique.

11. Which of the following measures of dispersion is not effected by changes in the smallest and the largest observations?

(A) Range.
(B) Mean Deviation about Median.
(C) Standard Deviation.
(D) Inter Quartile Range.

12. The number of red balls in a selection of 12 balls from a bag containing 20 red and 40 blue balls is a

(A) Negative Binomial random variable.

(B) Poisson random variable.

(C) Geometric random variable.

(D) Hypergeometric random variable.

13. The 5^{th} heads showed up in the 13^{th} toss of a coin.Some random variable is observed,which of the following is it?

(A) A Geometric random variable , observed to be 13.

(B) A Geometric random variable , observed to be 5.

(C) A Negative Binomial random variable , observed to be 13.

(D) A Negative Binomial random variable , observed to be 5.

14. The probability of success in a Bernoulli trial is 1/3 , the probability of getting more than $15,000$ successes in $30,000$ such independent trials

(A) can be approximated as a Normal probability.

(B) can be approximated as a Hypergeometric probability.

(C) can be approximated as a Poisson probability.

(D) can not be approximated at all.

15. $X_1 \sim B(10, 1/2)$, $X_2 \sim B(10, 1/3)$ and $X_1 \sim B(10, 3/4)$, so

(A) $V(X_1) > V(X_3) > V(X_2)$.

(B) $V(X_3) > V(X_2) > V(X_1)$.

(C) $V(X_1) > V(X_2) > V(X_3)$.

(D) $V(X_2) > V(X_3) > V(X_1)$.

16. The heights of adult males are normally distributed with mean $165cm.$ and Standard deviation $10cm.$, if a proportion p of the adult males have heights greater than $175cm.$, then the proportion of people taller than $155cm.$ is

(A) p. (B) $1 - p$.

(C) $\frac{p}{2}$ (D) $1 - \frac{p}{2}$

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17. Z_1 and Z_2 are independent Standard Normal random variables,then $Cov.(Z_1 + Z_2, Z_2 - Z_1)$ is equal to

(A) 0. (B) 2.

(C) −2. (D) 1.

18. The correlation coefficient between two random variables X and Y is -0.7 , it means that

(A) If X takes the value 2 , then $Y = -1.4$.

(B) If Y takes the value 2 , then $X = -1.4$.

(C) X and Y are independent random variables.

(D) By and large changes in X and Y happen in opposite directions.

19. In a class of 60 students , 10 got 40% , 25 got 50% , 10 got 60% , 5 got 70% , what can you say about the average marks of the remaining students if the class average is more than 58%?

(A) It is less than 55%.

(B) It is equal to 80%.

(C) It is more than 80% but less than 90%.

(D) It is more than 90%.

20. The regression coefficient of Y on X is β_1 and the regression coefficient of X on Y is β_2, then the correlation coefficient between X and Y is

(A) the Arithmetic mean of the two regression coefficients.

(B) the Geometric mean of the two regression coefficients.

(C) the Harmonic mean of the two regression coefficients.

(D) not any of the above.

21. Based on a random sample of size n, T_n is an unbiased estimator for a parameter θ ,and $V(T_n)$ is equal to θ^2 , an unbiased estimator for θ^2 based on the same sample is

(A) T_n^2. (B) $\frac{T_n^2}{2}$

(C) $T_n^2 + T_n$. (D) $2T_n^2$..

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22. In hypothesis testing , type-1 error α occurs when

(A) the sample is such that it falls in the critical region when the null hypothesis is true.

(B) the sample is such that it falls in the critical region when the null hypothesis is false.

(C) the sample is such that it falls in the critical region when the alternate hypothesis is true.

(D) the sample is such that it falls in the critical region when neither the null nor the alternate hypothesis is true.

23. The product of 5 positive numbers is 32 , then their sum

(A) is at most 5.

(B) is at least 10.

(C) is equal to 5.

(D) is none of the above.

24. The value of the integral $\int_0^\infty e^{-\frac{x^2}{2}}\, dx$ is

(A) $\sqrt{\pi}$. (B) $\frac{\sqrt{\pi}}{4}$.

(C) $\sqrt{2\pi}$. (D) $\sqrt{\frac{\pi}{2}}$.

25. The negation of the statement "Ashok did not eat at least one of the items " is

(A) Ashok ate at all the items.

(B) Ashok ate at exactly one item.

(C) Ashok ate at least one item.

(D) Ashok ate at most one item.

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Part-B

- Each question carries 3 marks.
- Questions (26)-(37) have more than one correct option.3 marks will be awarded only if all the correct options are marked in the OMR sheet, otherwise ZERO marks will be credited.
- Questions (38)-(50) have only one correct option. Find the correct answers and mark on the OMR sheet. Correct answers (marked in OMR sheet) to a question get 3 marks and ZERO otherwise.

26. The events A, B and C are mutually independent , then

(A) $P(A \cap B) = P(A)P(B)$, $P(A \cap C) = P(A)P(C)$, $P(B \cap C) = P(B)P(C)$.

(B) $P(A \cap (B \cap C)) = P(A)P(B \cap C)$.

(C) $P(C \cap (A \cup B) = P(C)P(A \cup B)$.

(D) $P(C \cap (A^c \cup B) = P(C)P(A^c \cup B)$.

27. For two non-constant real valued random variables X and Y and $V(X+Y) = V(X-Y)$, it means

(A) X and Y are independent.

(B) X and Y are uncorrelated.

(C) neither $V(X+Y)$ nor $V(X-Y)$ is zero.

(D) $X+Y$ can be some constant c.

28. In simple random sampling of n objects without replacement from a finite population of N distinct objects

(A) every member of the population has the same probability of being selected.

(B) the probability of a population member being selected in the sample depends on the number of draws in which it did not get selected.

(C) every collection of n objects has the same probability of being selected.

(D) some collections have a higher probability of being selected.

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29. The probability density function of a random variable X is

$$f_X(x) = \begin{cases} \frac{15}{4}x^2(1-x^2) & -1 < x < 1 \\ 0 & o.w \end{cases}$$

(A) The expected value of X does not exist.

(B) The median of X is 0.

(C) The expected value of X is 0.

(D) The variance of X is less than 1.

30. Suppose X is a Poisson random variable with parameter θ ,then

(A) The Minimum Variance Unbiased Estimator of θ is X.

(B) The Minimum Variance Unbiased Estimator of θ is $-X$.

(C) The Minimum Variance Unbiased Estimator of θ is $\sqrt{X}$.

(D) an unbiased estimator for $\frac{1-e^{-\theta}}{\theta}$ is $\frac{1}{X+1}$.

31. X is uniformly distributed over the interval $(2, 12)$ and Y is normally distributed with $E(Y) = E(X)$ and $V(Y) = V(X)$, further for $Z \sim N(0,1)$, $Pr(Z > 1.25) = 0.1$, then

(A) $Pr(2 < X \leq 6) = Pr(7 < X \leq 11)$ and $Pr(2 < Y \leq 6) = Pr(7 < Y \leq 11)$.

(B) X and Y are symmetrically distributed around their expected values.

(C) $Pr(X > 12) = 0$ whereas $Pr(Y > 12) > 0$.

(D) $Pr(11 < X \leq 12) > Pr(11 < Y \leq 12)$.

32. Suppose $0 \leq p_i \leq 1$, $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} p_i = 1$, then

(A) $\sum_{i=1}^{n} a_i^2 p_i < (\sum_{i=1}^{n} a_i p_i)^2$ for every collection of real numbers $\{a_1, a_2, \ldots, a_n\}$

(B) $\sum_{i=1}^{n} a_i^2 p_i \geq (\sum_{i=1}^{n} a_i p_i)^2$ for every collection of real numbers $\{a_1, a_2, \ldots, a_n\}$

(C) $\sum_{i=1}^{n} a_i^2 p_i = (\sum_{i=1}^{n} a_i p_i)^2$ if and only if $a_1 = a_2 = \ldots = a_n$

(D) There exist some real numbers $a_1, a_2, \ldots, a_n$ all of them not equal such that $\sum_{i=1}^{n} a_i^2 p_i = (\sum_{i=1}^{n} a_i p_i)^2$.

33. Whenever two real,non-constant finite valued random variables X and Y are independent, then

(A) $Cov.(X,Y) = 0$

(B) $E(X|Y = y_1) = E(X|Y = y_2)$, for any $y_1 \neq y_2$.

(C) X^2 and Y^2 need not be independent.

(D) $V(XY) = V(X)V(Y)$

34. $P(A_1) = 1/3$, $P(A_2) = 1/3$, $P(A_3) = 1/4$, then

(A) $P(A_1 \cup A_2 \cup A_3) > 11/12$.

(B) $P(A_1 \cup A_2) \leq 2/3$.

(C) $P(A_1^c \cap A_2^c \cap A_3^c) < 1/12$.

(D) $P(A_1 \cup A_3^c) > 1/2$.

35. Identify the correct statements using Chebychev's inequality

(A) For a random variable X, $E(X) = 0$ and $V(X) = 16$, then $Pr(-10 < X < 10)$ is not more than 0.7

(B) For a random variable X, $E(X) = 0$ and $V(X) = 16$, then $Pr(-10 < X < 10)$ could be 0.9.

(C) For a random variable X, $E(X) = 0$, if $Pr(-12 < X < 12)$ is 0.7, then we can say nothing about $V(X)$.

(D) For a random variable X, $E(X) = 0$, if $Pr(-12 < X < 12)$ is 0.7, then $V(X)$ could be 45.

36. Of all the 6! possible arrangements of 6 distinct objects : $o_1, o_2, \ldots, o_6$ in a row

(A) o_6 appears anywhere on the left of o_4 in 240 arrangements.

(B) o_2 appears anywhere on the right of o_1 in 360 arrangements.

(C) o_1 and o_3 appear next to each other in 240 arrangements.

(D) o_6 appears on the left of all the other objects in 240 arrangements.

37. The columns of real $n \times n$ matrix B are linear combinations of columns of another real $n \times n$ matrix A,so

(A) $rank(B) \leq rank(A)$.

(B) There exists a real $n \times n$ matrix C such that $AC = B$.

(C) If $det.A = 0$, so is $det.B$.

(D) If $\mathbf{x}$ is a non-zero vector satisfying $A^T\mathbf{x} = \mathbf{0}$, then $B^T\mathbf{x}$ is also equal to 0.

38. A bag contains 10 red (all alike) and 10 blue (all alike) balls, balls are drawn one after the other without replacing , the probability that the second ball drawn is red is

(A) 10/19. (B) 9/19.

(C) 1/2. (D) 11/20.

39. 4, 3, 5, 2, 6 are 5 observations of the $B(10, p)$ random variable an unbiased estimate of $(1+p)^{10}$ is

(A) $(1.4)^{10}$. (B) 24.8.

(C) $(\frac{3}{2})^{10}$. (D) $(\frac{4}{3})^{10}$.

40. 1 is added to the smallest and subtracted from the largest of $n(> 2)$ distinct positive integers, let m_1, m_2 and s_1, s_2 denote the means and standard deviations of the original and the new set of integers respectively, then

(A) $m_2 = m_1$ and $s_2 = s_1$.

(B) $m_2 = m_1$ and $s_2 < s_1$.

(C) $m_2 > m_1$ and $s_2 = s_1$.

(D) $m_2 = m_1$ and $s_2 > s_1$.

41. 5, 6, 4, 3, 2 are 5 observations of a Poisson random variable with parameter λ , the unbiased estimate of λ^2 is

(A) 10. (B) 5.

(C) 19. (D) 14.

42. The joint probability mass function of two random variables X and Y is

$$Pr(X = i, Y = j) = \frac{c}{ij} \text{ for } i = 1, 2, 3 \text{ and } j = 1, 2 \text{ , where } c > 0 \text{ , then}$$

the conditional expectation of X , given that $Y = 2$, that is $E(X|Y = 2)$ is in the interval

(A) $(0, 1]$. (B) $(1, 2]$.

(C) $(2, 2.5]$. (D) $(2.5, 2.75]$.

43. The Probability mass function of a random variable X is $Pr(X = x) = (\frac{\theta}{1+\theta})^x(\frac{1}{1+\theta})^{1-x}$, $x = 0, 1$ where $\theta > 0$,a sample of 10 observations of this random variable was observed to be 0, 1, 0, 0, 0, 1, 0, 1, 0, 1 the Maximum Likelihood Estimate of θ is

(A) 1. (B) 2/3.

(C) 4/3. (D) 5/3.

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44. $X_1, X_2, \ldots, X_n$ is a random sample from the uniform population over $(0, \theta]$, now the sample contains some information about θ, this information is also contained in

(A) the sample mean: $\overline{X} = \frac{X_1+X_2+\ldots+X_n}{n}$.

(B) the maximum in the sample : $X_{(n)} = max.(X_1, X_2, \ldots, X_n)$.

(C) the minimum in the sample : $X_{(1)} = min.(X_1, X_2, \ldots, X_n)$.

(D) the median in the sample.

45. A bank opens at 9.00AM., the time X in minutes, from then on till the arrival of the first customer is an exponential random variable and $Pr(X > 6) = 1/64$, then the expected time till the first arrival

(A) is less than half a minute.

(B) is more than 2 a minutes.

(C) is more than 1 minute , but less than 2 minutes.

(D) can not be determined from the data given.

46. The variance of the first n odd natural numbers is

(A) $\frac{n^2-1}{12}$. (B) $\frac{n^2-1}{4}$.

(C) $\frac{n^2-1}{3}$. (D) $\frac{n^2-1}{6} + 1$.

47. The first and second moments of which of the following random variables can be equal?

(A) Bernoulli random variable with parameter p.

(B) Binomial random variable with parameters n, p.

(C) Poisson random variable with parameter λ.

(D) Geometric random variable with parameter p.

48. Given below are measurements for six individuals on two random variables X and Y

X	4	10	12	16	18	24
Y	1.5	4.5	5.5	7.5	8.5	11.5

We can say that the correlation coefficient between X and Y is

(A) 1. (B) -1.

(C) 0. (D) 0.5.

49. The value of $\sum_{j=0}^{n} \binom{2n}{2j}$ is equal to

(A) 2^n. (B) 4^n.

(C) 4^{n-1}. (D) 2^{2n-1}.

50. The proportion of bacteria that will die within one day of treatment is a random variable X whose probability density function is

$$f(x) = \begin{cases} cx^4(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

the expected proportion of bacteria that will die within one day of treatment is

(A) 1/3. (B) 3/5.

(C) 5/7. (D) 7/9.

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