Booklet Code A

Entrance Examination (June 2011)

Master of Computer Applications (MCA)

Time: 2 Hours   **Max. Marks: 100**

Hall Ticket Number:

INSTRUCTIONS

1. (a) Write your Hall Ticket Number in the above box AND on the OMR Sheet.

   (b) Fill in the OMR sheet, the **Booklet Code A** given above at the top left corner of this sheet. Candidates should also read and follow the other instructions given in the OMR sheet.

2. All answers should be marked clearly in the OMR answer sheet only.

3. This objective type test has two parts: Part A with 25 questions and Part B with 50 questions. Please make sure that all the questions are clearly printed in your paper.

4. Every correct answer in **Part A** carries **2 (two) marks** for every wrong answer **0.66** mark will be deducted.

5. Every correct answer in **Part B** carries **1 (one) mark** and for every wrong answer **0.33** mark will be deducte

6. Do not use any other paper, envelope etc for writing or doing rough work. All the rough work should be done in your question paper or on the sheets provided with the question paper at the end.

7. During the examination, anyone found indulging in copying or have any discussions will be asked to leave the examination hall.

8. Use of non-programmable calculator and log-tables is allowed.

9. Use of mobile phone is NOT allowed inside the hall.

10. Submit both the question paper and the OMR sheet to the invigilator before leaving the examination hall.

V-09

Part A

1. The length of the perimeter of a right triangle is 60 units. The length of the perpendicular to the hypotenuse is 12 units. The sum of the lengths of the two sides apart from the hypotenuse is

   A. 35

   B. 120

   C. 30

   D. 125

2. In a number system, the product of 44 and 11 is given by the number 1034 in that system. The number 3111 of this system, when converted to the decimal number system becomes:

   A. 1086

   B. 406

   C. 213

   D. 691

3. Manufacturer of a tennis ball announced that their new ball is the best in the market. If one were to drop it from any height, it would only rise to the extent of 10% of the height from which it was dropped. If the ball was dropped from 45 feet, how much would the ball travel before coming to rest?

   A. 45 ft

   B. 50 ft

   C. 55 ft

   D. 49.5 ft

4. There are 7 sets of numbers $S_1, S_2, \ldots, S_7$ such that $S_1 = \{a\}$, $S_2 = \{b, c, d\}$, $S_3 = \{c, d, e\}$, $S_4 = \{f, g\}$, $S_5 = \{h\}$, $S_6 = \{i, j, k\}$ and $S_7 = \{j, k, l\}$. If $a - l$ represent numbers in ascending order, and we need to create a new list M such that it contains exactly one element from each set S_i in strict sequence and the elements are in *strict* ascending order, how many different such lists can be created?

   A. 162

   B. 32

   C. 36

   D. 72

Answer questions 5 and 6 using the following three operations (or steps) only (i.e., no others):

SPILL: will spill water down the drain (out of system)
FILL: fill one jug full, with water from tap
POUR(a,b): pour from jug a to jug b until either jug a is empty or jug b is full

Assume that the jugs have no measurements on them and that all jugs are initially empty. Always determine the **minimal** number of steps to solve a problem

V-09

5. Given a three-litre jug and a four-litre jug, how many steps are required to reach the goal of exactly two litres of water in the four-litre jug?

A. 6 steps

B. 4 steps

C. 5 steps

D. 3 steps

6. Given a five-litre jug and an eight-litre jug, how many steps are required to reach the goal of exactly two litres of water in the five-litre jug?

A. 3 steps

B. 4 steps

C. 5 steps

D. 6 steps

7. In the figure below, Gears A and B have a diameter of 12 cm and 24 teeth each, Gear B has a diameter of 3 cm and 8 teeth, while Gear D has a diameter of 6 cm and 24 teeth. If Gear A is turned clockwise (as shown by the arrow) at 16 rotations/minute (rpm), which of the following is TRUE about Gear D?
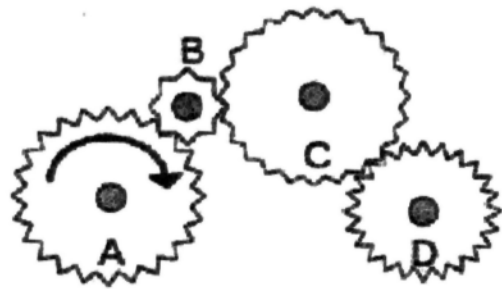
A. rotates at 16 rpm in counter-clockwise direction

B. rotates at 16 rpm in clockwise direction

C. rotates at 32 rpm in counter-clockwise direction

D. rotates at 24 rpm in counter-clockwise direction

8. A gardener was planting a new orchard. The young trees were arranged in rows so as to form a square and it was found that there were 146 trees unplanted. To enlarge the square by an extra row each way she had to buy 31 additional trees. How many trees were there in the orchard when it was finished?

A. 7921

B. 256

C. 7056

D. None of the above

9. Let us look at constructing a *house of cards*. The smallest house consists of 2 cards placed on a level surface. A house with two floors needs

7 cards (see Figure below). What is the maximum number of floors in a house constructed using a single deck of 52 cards (some cards may be left over)?

A. 3

B. 4

C. 5

D. 6

Answer 10 using the following logic:

X, Y and Z are playing a game. There are a certain number of matchsticks on a table. Each person has to pick up one matchstick and at most two matchsticks. The person who picks up the last matchstick loses.

X, Y and Z play in that order one after the other, starting with X, till all the match sticks are picked up. Each person plays such that, to the extent possible, he does not lose even if the other two players collude.

10. How many match sticks should X and Y respectively pick, in their first chance, so that Z is certain to lose if

X and Y play with common objective of making Z lose? There were 7 matchsticks to start with.

A. (2,1)

B. (1,2)

C. (1,1)

D. (2,2)

11. In a remote army post there are four clubs. The membership lists reveal that
(i) Each soldier is a member of two clubs
(ii) Every set of two clubs has only one member in common
How many soldiers are there at this post?

A. 5

B. 6

C. 8

D. 11

12. A wealthy woman had three servants – S_1, S_2 and S_3 – was estimating that the amount of grain in her storeroom would suffice for S_1 and S_2 for 45 days. S_1 and S_3 would eat all the grain in 60 days. It would take S_2 and S_3 90 days to eat it all. Now if she were to feed S_1, S_2 and S_3 together, how long would it have taken them to eat all the grain?

A. 40 days

B. 65 days

C. 180 days

D. None of the above

13. $100 != 1 \times 2 \times 3 \times \ldots \times 100$ ends exactly in how many zeroes?

    A. 24

    B. 10

    C. 11

    D. 21

14. A man has 10 pockets. He wants to put different number of gold coins into each pocket. He went to a goldsmith and explained this desire to him. The goldsmith sold him 30 gold coins and asked him to come again the next day. What is the minimum number of additional gold coins the man needs to buy to fulfill his desire?

    A. 18

    B. 14

    C. 16

    D. 15

15. Let a and b be two positive integers. The number of factors of $5^a 7^b$ are

    A. $2^{(a+b)}$

    B. $a + b + 2$

    C. $ab + 1$

    D. $(a + 1)(b + 1)$

16. There are 13 white and 15 black marbles placed in a box. We remove two marbles from the box at every step. If they have a different colour, put the white marble back in the box. If they have the same colour, we put an additional black marble in the box (there are 28 black marbles available outside the box). Then the following situation occurs just before the last step.

    A. Two white marbles are left

    B. Two black marbles remain

    C. One black and one white marble are left

    D. Either two white or two black marbles can remain

17. Arithmetic mean of two positive numbers is $18\frac{3}{4}$ and their geometric mean is 15. The larger of the two numbers is

    A. 30

    B. 20

    C. 24

    D. None of the above

Question 18 is based on the following information:

  a. Eight trees, Mango, Guava, Pomegranate, Lemon, Banana, Raspberry and Apple are planted in two rows, 4 in each row facing North and South.

  b. Lemon is between Mango and Apple but just opposite Guava.

  c. Banana is at the end of a line and is just next to the right of Guava or either Banana tree is just after Guava.

V-09

d. Raspberry tree which is at one end of a line is just diagonally opposite to Mango tree.

18. Which of the following statements is definitely true?

   A. Papaya tree is just near Apple tree.

   B. Apple tree is just next to Lemon tree.

   C. Raspberry tree is either to the left of Pomegranate or after.

   D. Pomegranate is diagonally opposite to the Banana tree.

Answer Question 19 using the following information:
In a quiz with 8 teams, correct answer for a direct question gets 10 points, and correct answer for an indirect question (i.e., a question passed down as the other teams could not answer) gets 5 points. There are 40 questions in the round and they are asked in strict sequence starting from the first question to Team 1, the second to Team 2 and so on.

19. What is the maximum *sum* or *aggregate* score that can be achieved by all the teams together?

   A. 200

   B. 400

   C. 1800

   D. 300

20. An ant, located in a square field is 13 meters from one of the corner posts of the field, 17 meters from the corner post diagonally opposite that one and 20 meters from the third corner post. Find the area of the field. Assume that the land is flat.

   A. 89 m^2

   B. 231 m^2

   C. 369 m^2

   D. 169 m^2

21. Many people consider numbers whose digits add up to '9' as *lucky* numbers. For example, '2781' is considered *lucky* as its digits add up to '18' which in turn results in '9.' How many such *lucky* numbers are available for vehicle numbers? Remember that a vehicle number may be 1, 2, 3 or 4 digits long.

   A. 999

   B. 900

   C. 1111

   D. 1089

22. A circle of radius 1 is inscribed in an equilateral triangle of suitable size. Then three more circles are inscribed between the first circle and the two sides of the triangle near each vertex. The process continues indefinitely with progressively smaller circles. What is the sum of radii of all the circles?

   A. 2.0

B. 1.5

C. 3.0

D. 2.5

Answer Qestions 23-25 using the logic given below:
The following details regarding the age, weight and height of six different individuals are available. The six persons are A, B, C, D, E and F.

(a) B is the oldest person and taller than C but shorter than A and heavier than D.

(b) F is not heavier than C but heavier than E and older than D, who is older than A, who is the youngest person.

(c) A is the heaviest person and shorter than F, who is shorter than D.

(d) C is taller and older than E but not heavier than D.

23. Who is the tallest person?

A. C

B. F

C. A

D. D

24. If at most two persons are older than E, then how many persons are younger than D?

A. 4

B. 1

C. 3

D. 2

25. Which of the following cannot be the order of the names of the persons from the oldest to the youngest?

A. B, F, D, C, E , A

B. B, C, F, D, E, A

C. B, F, C, A, D, E

D. B, C, F, E, D, A

Part B

26. Choose the value of X that satisfies the equality

$$(2201.375)_{10}+X = (1084.2)_{16}+(5272.7)_8$$

A. $(10D.C)_{16}$

B. $(35.6)_8)$

C. $(11101.110)_2$

D. $(11101.101)_2$

27. If A is a 2×2 real matrix such that $A - 3I$ and $A - 4I$ are not invertible, then A^2 is

A. $12A - 7I$

B. $7A - 12I$

C. $7A + 10I$

D. $12I$

28. Let the probability density function

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$

The value of k is

V-09

A. $\frac{1}{8}$

B. $\frac{3}{5}$

C. $\frac{2}{5}$

D. $\frac{3}{8}$

29. If A and B are 3×3 matrices with $|A|=4$ and $|B|=3$, which of the following is generally false?

   A. $3|B|=9$

   B. $2|A|=32$

   C. $|AB|=12$

   D. $|A+B|=7$

30. The region bounded by the graphs $y = x^2$ and $y = x + 2$ is revolved about the line $x = 3$. The resulting solid has a volume that can be computed using

   A. $\int_{-1}^{0} 2\pi(3+x)(x-2+x^2)dx$

   B. $\int_{-1}^{2} 2\pi(3-x)(x+2-x^2)dx$

   C. $\int_{0}^{1} 2\pi(3+x)(x+2+x^2)dx$

   D. $\int_{0}^{2} 2\pi(3-x)(x+2-x^2)dx$

31. For $a>1$, the value of $\int_{0}^{\pi} \dfrac{dx}{a^2 - 2a\cos x + 1}$ is

   A. $2\pi a$

   B. $\dfrac{\pi}{2a}$

   C. $\dfrac{\pi}{a^2-1}$

D. 0

32. A line makes angles α, β, γ and δ with the four diagonals of a cube. Then the sum

$$cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta$$

is

   A. $\frac{4}{3}$

   B. 0

   C. $\frac{1}{3}$

   D. 1

33. Consider the lines given by $(x = a_1z + b_1, y = c_1z + d_1)$ and $(x = a_2z + b_2, y = c_2z + d_2)$. The condition by which these lines would be perpendicular is given by
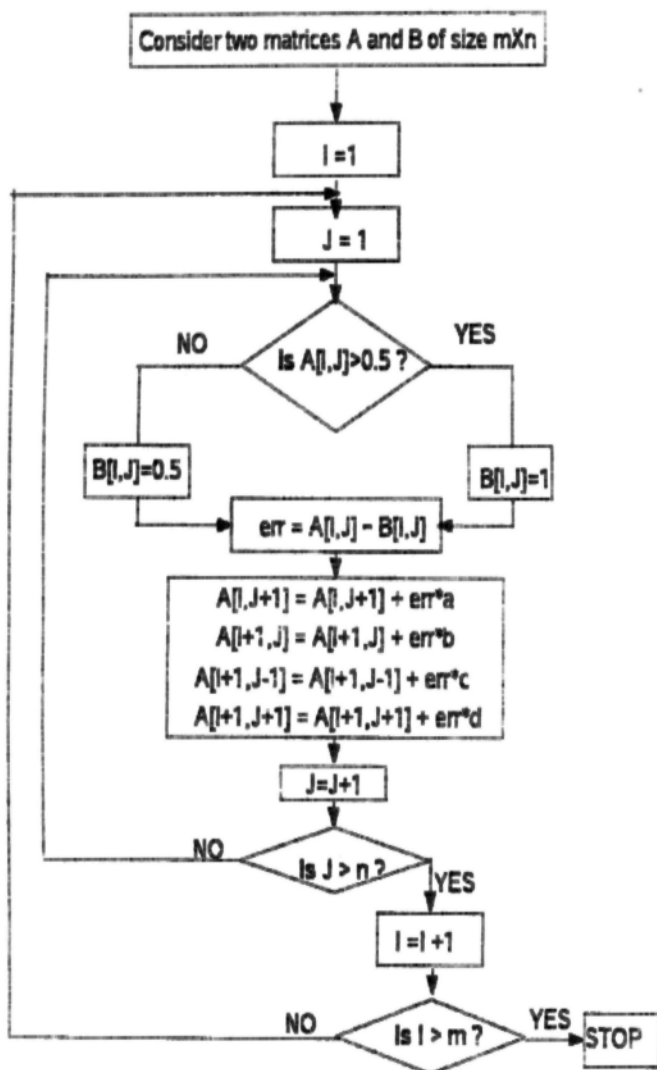
   A. $a_1c_1 - a_2c_2 + 1 = 0$

   B. $a_1c_1 + a_2c_2 - 1 = 0$

   C. $a_1a_2 - c_1c_2 = 1$

   D. $a_1a_2 + c_1c_2 + 1 = 0$

Consider the flow chart given below and answer questions 34 – 36 based upon it.

V-09

```
┌──────────────────────────────────────┐
│ Consider two matrices A and B of size mXn │
└──────────────────────────────────────┘
                    │
                 ┌─────┐
                 │ I=1 │
                 └─────┘
                    │
                 ┌─────┐
                 │ J=1 │
                 └─────┘
                    │
      NO       ◇ Is A[I,J]>0.5 ? ◇      YES
   ┌────────────┘              └────────────┐
┌─────────┐                            ┌────────┐
│B[I,J]=0.5│                           │ B[I,J]=1 │
└─────────┘                            └────────┘
        └──→ ┌──────────────────┐ ←──────┘
             │ err = A[I,J] – B[I,J] │
             └──────────────────┘
                    │
      ┌──────────────────────────────────┐
      │ A[I,J+1] = A[I,J+1] + err*a        │
      │ A[I+1,J] = A[I+1,J] + err*b        │
      │ A[I+1,J-1] = A[I+1,J-1] + err*c    │
      │ A[I+1,J+1] = A[I+1,J+1] + err*d    │
      └──────────────────────────────────┘
                    │
                ┌───────┐
                │ J=J+1 │
                └───────┘
                    │
      NO    ◇ Is J > n ? ◇   YES
                    │
                ┌───────┐
                │ I=I+1 │
                └───────┘
                    │
      NO    ◇ Is I > m ? ◇   YES  ┌──────┐
                                  │ STOP │
                                  └──────┘
```

34. If A is a 3×3 matrix, what is $B[2,2]$? where $a = 1.2, b = 3, c = 0.5, d = 5$.

$$A = \begin{bmatrix} 3 & 0 & 5 \\ 15 & 2 & 1 \\ 2 & 4 & 5 \end{bmatrix}$$

A. 29.74

B. 52.78

C. 42.556

D. 15.68

35. If A is a 3×3 matrix, the number of times the elements in A will be modified are

A. 20

B. 9

C. 10

D. 21

36. If A is a 5×5 matrix, the number of times the elements of A are modified will be

A. 73

B. 70

C. 72

D. 74

37. The image P' of the point $P(p,q,r)$ in the plane $2x + y + z = 6$

A. $(p, q, -r)$

B. $\frac{1}{3}(12 - p - 2q - 2r), \frac{1}{3}(6 - 2p + 2q - r), \frac{1}{3}(6 - 2p - q + 2r)$

C. $\frac{1}{3}(6 - 2p - q - 2r), \frac{1}{3}(12 - p - 2q - r), \frac{1}{3}(6 - 2p - q + 2r)$

D. $\frac{1}{3}(6 - 2p + 2q - r), \frac{1}{3}(6 - 2p + 2q - r), \frac{1}{3}(12 - p - 2q - 2r)$

38. The image of the line from the point P given in Question 37 and it's reflection P' about the plane $2x+y+z = 6$ is given by

A. $\frac{3x - 1}{4} = \frac{3y - 5}{2} = \frac{3z - 8}{-1}$

B. $\frac{x - 1}{2} = \frac{y - 2}{1} = \frac{z - 3}{4}$

V-09

C. $\dfrac{x-1}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{2}$

D. $\dfrac{3x-1}{2}=\dfrac{3y-1}{3}=\dfrac{z-8}{-1}$

39. In 2's complement notation A=00100101, B=11110001 and X=A+B. If 2's complement arithmetic is used to perform the addition what is the Gray code of X?

   A. 00010110

   B. 00011011

   C. 00011101

   D. 11101010

40. If $\sqrt{1552}=34$, then the base of the number system is

   A. 9

   B. 6

   C. 8

   D. 7

41. Evaluate $\displaystyle\int \dfrac{1}{1+x^2}dx$

   A. $-\pi$

   B. $\dfrac{-\pi}{2}$

   C. $\dfrac{\pi}{2}$

   D. π

42. Consider the region bounded by the graphs $y=e^x$, $y=0, x=1$ and $x=t$, where $t<1$. The area of this region is atmost

   A. unbounded

   B. e

C. 0

D. 1^e

43. The value of p for which the matrix

$$A=\begin{bmatrix} 3 & p & p \\ p & 3 & p \\ p & p & 3 \end{bmatrix}$$

has rank 2 is

   A. $\dfrac{3}{2}$

   B. $\dfrac{1}{2}$

   C. $-\dfrac{3}{2}$

   D. 1

44. Let $A=2i-3j+k$ and $B=-i+2j+k$ be two vectors. The vector perpendicular to both A and B having length 10 is

   A. $\dfrac{-50i-30j+70k}{\sqrt{83}}$

   B. $\dfrac{50i+30j-70k}{\sqrt{83}}$

   C. Both A and B

   D. None of the above

45. Let $f_1(x)=e^x$, $f_2(x)=e^{f_1(x)}$, $f_3(x)=e^{f_2(x)}$, ... and in general $f_{n+1}(x)=e^{f_n(x)}$ for any $n\geq 1$. For any fixed value of n, the value of $\dfrac{d}{dx}f_n(x)$ is

   A. $f_n(x)$

   B. $f_n(x)f_{n-1}(x)f_{n-2}(x)\ldots f_2(x)f_1(x)$

   C. $f_n(x)\ f_{n-1}(x)$

   D. $f_n(x)f_{n-1}(x)f_{n-2}(x)\ldots f_2(x)f_1(x)e^x$

V-09

46. An unprepared student takes a five question true-false exam and guesses every answer. What is the probability that the student will pass the exam if at least four correct answers is the passing grade?

    A. $\frac{3}{16}$

    B. $\frac{5}{32}$

    C. $\frac{1}{32}$

    D. $\frac{1}{8}$

Answer questions 47 and 48 using the following text:
In a country club, 60% of the members play tennis, 40% play shuttle and 20% play both tennis and shuttle. When a member is chosen at random,

47. What is the probability that she plays neither tennis nor shuttle?

    A. 0.8

    B. 0.2

    C. 0.5

    D. 0.4

48. If she plays tennis, what is the probability that she also plays shuttle?

    A. $\frac{2}{3}$

    B. $\frac{2}{5}$

    C. $\frac{1}{3}$

    D. $\frac{1}{2}$

49. If E is the event that an applicant for a home loan is employed C is the event that she possesses a car and A is the event that the loan application is approved, what does $P(A|E \cap C)$ represent in words?

    A. Probability that the loan is approved, if she is employed and possesses a car

    B. Probability that the loan is approved, if she is either employed or possesses a car

    C. Probability that the loan is approved, if she is neither employed nor possesses a car

    D. Probability that the loan is approved and she is employed, given that she possesses a car

In questions 50 and 51, for sets X and Y, $X \triangle Y$ is defined as $X \triangle Y = (X - Y) \triangle (Y - X)$

50. If $P = \{1,2,3,4\}$, $Q = \{2,3,5,8\}$, $R = \{3,6,7,9\}$ and $S = \{2,4,7,10\}$ then $(P \triangle Q) \triangle (R \triangle S)$ is

    A. $\{4,7\}$

    B. $\{1,5,6,10\}$

    C. $\{1,2,3,5,6,8,9,10\}$

    D. None of the above

51. If X, Y, Z are any three subsets of U, then the subset of U consisting of elements which belong to exactly two of the sets X, Y, Z is

    A. $(X \cap Y) \cup (Y \cap Z) \cup (Z \cap X)$

    B. $(X \cap Y) \triangle (Y \cap Z) \triangle (Z \cap X)$

    C. $((X \cup Y) \cup Z) - ((X \triangle Y) \triangle Z)$

D. None of the above

52. The rank of the matrix A is

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}$$

A. 3

B. 2

C. 4

D. 5

53. If the regression coefficient of X on Y is $-\frac{1}{6}$ and that of Y on X is $-\frac{3}{2}$. What is the correlation coefficient between X and Y?

A. 0.5

B. -0.5

C. 0.25

D. -0.75

54. A national achievement test is administered annually to 3rd graders. The test has a mean score of 100 and a standard deviation of 15. If a person's *z-score* (normalized score) is 1.2, what is the score of the person on the test?

A. 112

B. 88

C. 100

D. 118

55. A company sells baseball cards in packages of 100. Three types of players are represented in each package-rookies, veterans and all-stars. The company claims that 30% of the cards are rookies, 60% are veterans and 10% are all-stars. Cards from each group are randomly assigned to packages. Suppose you bought a package of cards and counted the players from each group. What method would you use to test the company's claim that 30% of the cards are rookies, 60% are veterans and 10% are all-stars.

A. Chi-square goodness of fit test

B. Chi-square test for homogeneity

C. One-sample t test

D. Matched pairs t-test

56. If θ is the angle between the diagonal of a cube and the diagonal of the base of the cube then $\tan\theta$ is

A. $\frac{\sqrt{2}}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{3}$

D. $\frac{\sqrt{2}}{2}$

57. The values of r for which the differential equation $x^2 y'' + 4xy' + 4y = 0$ has a solution of the form $y = x^r$

A. 1

B. -1

C. -2

D. None

58. A curve given in polar form as $r = a(cos(\theta) + sec(\theta))$ can be written in Cartesian form as

    A. $x(x^2 + y^2) = a(2x^2 + y^2)$

    B. $y^2 = a(x^2 + \frac{1}{x^2})$

    C. $y = a(x + \frac{1}{x})$

    D. $y = atan\theta + x$

59. If $sin(x) = \frac{4}{5}$, then the value of $tan(\frac{x}{2})$ is

    A. $\frac{1}{2}$ or 2

    B. $\frac{1}{2}$ or -2

    C. $\frac{3}{4}$ or -2

    D. $\frac{3}{4}$ or 2

60. The straight line through the point (-1,3,3) pointing in the direction of the vector (1,2,3) hits the xy plane at the point

    A. (2,-1,0)

    B. (-2,1,0)

    C. (1,3,0)

    D. never

61. The $n \times n$ matrix P is idempotent if $P^2 = P$ and orthogonal if $P'P = I$. Which of the following is false?

    A. If P and Q are idempotent $n \times n$ matrices and $PQ = QP = 0$, then $P + Q$ is idempotent

    B. If P is idempotent then $-P$ is idempotent

C. If P and Q are orthogonal $n \times n$ matrices then PQ is orthogonal

D.
$$P = \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

62. Suppose $f(x)$ is defined by $f(x) = min(x, x^2) \ \forall \ x \in \Re$. Then which of the following statements is true?

    A. f is not differentiable at exactly one point

    B. $f' \neq 1$ at any x $\Re$

    C. $f(x)$ is a discontinuous function

    D. $f(x)$ is not differentiable at exactly two points in $\Re$

63. A plane flies 810 miles from A to B with a bearing 75° East of North. Then it flies 648 miles from B to C with a bearing 32° East of North. The straight line distance from C to A is approximately

    A. 1358 miles

    B. 1558miles

    C. 1658 miles

    D. 1958 miles

64. Let A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) and D (x_4, y_4) be four points such that x_1, x_2, x_3, x_4 and y_1, y_2, y_3, y_4 are both in arithmetic progression. Then the area of the quadrilateral ABCD is
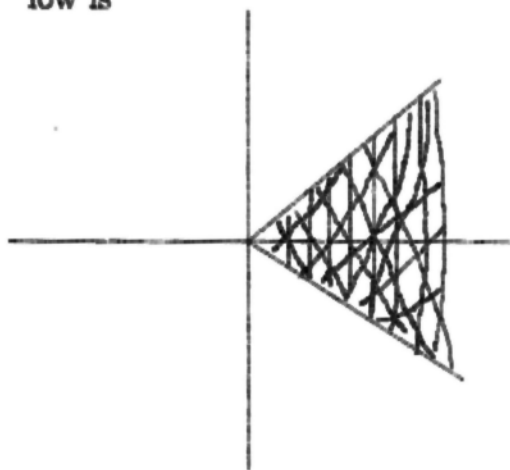
    A. 0

B. greater than 1

C. less than 1

D. Depends on the coordinates of A,B,C,D

65. Let

$$A = \begin{bmatrix} 4 & 1 & 0 \\ 2 & -1 & 2 \\ x & y & -1 \end{bmatrix}$$

be a 3×3 matrix. Let x and y be the values such that matrix A is singular. What is $x + y$?

A. 0

B. 3

C. $\frac{1}{2}$

D. 2

66. The relation that represents the shaded region in the figure given below is

A. $y \le x$

B. $|y| \le |x|$

C. $y \le |x|$

D. $|y| \le x$

67. Let $f : [0,1] \rightarrow [0,1]$ be a function that is twice differentiable in its domain, then the equation $f(x) = x$ has

A. no solution

B. exactly one solution

C. at least one solution

D. not enough data to say about number of solutions

68. The general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x + y^2}{2xy}$

A. $y^2 = (ln\,|\,x\,| + C)x$

B. $y = (ln\,|\,x\,| + C)x$

C. $y^2 = (ln\,|\,x\,| + C)$

D. $y = (ln|x| + C)x^2$

69. Let $f(x)$ be the function defined on the interval $(0,1)$ by

$$f(x) = \begin{cases} x(1 - x) & \text{if x is rational} \\ \frac{1}{2} - x(1 - x) & \text{if x is not rational} \end{cases}$$

then f is continuous at

A. no point in $(0,1)$

B. at exactly 2 points in $(0,1)$

C. at exactly one point in $(0,1)$

D. at more than 2 points in $(0,1)$

70. A cup of hot chocolate is initially at 170°F and is left in a room with ambient temperature of 70°F. Assume rate of cooling is proportional

to difference of current and ambient temperatures. Let rate constant be 0.2°F/minute. How long does it take for the chocolate to cool down to a temperature of 110°F?

A. 10

B. 5

C. $5 \ln\frac{5}{2}$

D. $\ln\frac{5}{2}$

71. If

$$y = \sin^{-1}\left(\frac{\sin(\alpha)\sin(x)}{1 - \cos(\alpha)\sin(x)} \right)$$

then

$$\frac{dy}{dx}\bigg|_{x=0}$$

A. 1

B. $\sin(\alpha)$

C. 0

D. $\cos(\alpha)$

72. Suppose $f(x) = \lceil x^2 \rceil - \lceil x \rceil^2$ where $\lceil x \rceil$ denotes the largest integer $\leq x$. Then which of the following statements is true?

A. $f(x) > 0 \ \forall \ x \in \Re$

B. $f(x)$ can be discontinuous at points other than the integral values of x

C. $f(x)$ is a monotonically increasing function

D. $f(x) \neq 0$ everywhere, except on the interval [0,1]

73. Let $\vec{p}$ and $\vec{q}$ be position vectors of P and Q respectively. The points R and S divide PQ internally and externally in the ratio 2:3 respectively. If OR $\perp$ OS then

A. 4p=9q

B. $4p^2=9q^2$

C. $9p^2=4q^2$

D. 9p=4q

74. If $|\vec{p}|+|\vec{q}|=\sqrt{13}$ and $|\vec{p}|-|\vec{q}|= 1$ and $|\vec{p}|= \sqrt{3}$, then the angle between $\vec{p}$ and $\vec{q}$ is

A. $\frac{4\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{6}$

D. $\frac{\pi}{6}$

75. The differential equation, whose solutions are all the circles in a plane, is given by

A. $(1+y')^2 y''' - 3y'y''^2 = 0$

B. $xy' + y = 0$

C. $(1+y')^2 y''' + 3y'y''^2 = 0$

D. $yy'' + y'^2 + 1 = 0$

V-09