J- 56

ENTRANCE EXAMINATION, FEBRUARY 2015

QUESTION PAPER BOOKLET

Ph.D. (PHYSICS)

Marks: 75

Time: 2.00 hrs.

Hall Ticket No.:

I. Please enter you **Hall Ticket Number on Page 1 of this question paper and on the OMR sheet** without fail.

II. Read carefully the following instructions:

1. This Question paper has 2 Sections: **Section A** and **Section B**
2. **Section A** consists of 25 objective type questions of one mark each.
   There is negative marking of 0.33 mark for every wrong answer.
   The marks obtained by the candidate in this Section will be used for resolving the tie cases.
3. **Section B** consists of 50 objective type questions of one mark each.
   There is no negative marking in this Section.
4. Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below

   100. (A) (B) (C) ●

5. Scientific Calculators are not permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
6. Hand over both question booklet and the OMR sheet at the end of the examination.

This book contains 24 pages

III. Values of physical constants:
$c = 3 \times 10^8$ m/s; $h = 6.63 \times 10^{-34}$ J.s; $k_B = 1.38 \times 10^{-23}$ J/K
$e = 1.6 \times 10^{-19}$ C; $\mu_o = 4\pi \times 10^{-7}$ Henry/m; $\epsilon_o = 8.85 \times 10^{-12}$ Farad/m

J-56

THIS PAGE IS LEFT BLANK INTENTIONALLY

THIS PAGE IS LEFT BLANK INTENTIONALLY

THIS PAGE IS LEFT BLANK INTENTIONALLY

J-56

SECTION - A

1. If a function $f(z) = u(x, y) + iv(x, y)$ of a complex variable $z = x + iy$ is analytic at a point $z = z_0$ then at that point the following conditions hold

   A. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$; $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$.

   B. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$; $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$.

   C. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$; $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial y}$.

   D. $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial x}$; $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial y}$.

2. The Fourier transform $F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, e^{-i\omega t} f(t)$ of a real function $f(t)$ necessarily satisfies(Note: $F^*(\omega)$ is the complex conjugate of $F(\omega)$.)

   A. $F(\omega) = F(-\omega)$.

   B. $F(\omega) = F^*(-\omega)$.

   C. $F(\omega) = -F^*(\omega)$.

   D. $F(\omega) = F^*(\omega)$.

3. The second-order algebraic surface given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$ represents

   A. an ellipsoid.

   B. a hyperboloid of one sheet.

   C. a hyperboloid of two sheets.

   D. a cone of second order.

4. A linear system is given by the relations $\dot{x} = -2x + \alpha y$, $\dot{y} = x + y$. The condition to be satisfied by for an arbitrary point in this linear system to be a saddle point is

   A. $\alpha > -2$.

   B. $-\frac{9}{4} < \alpha < -2$.

   C. $\alpha < -9/4$.

   D. $\alpha = -9/4$.

5. The ratio of the speed of an electron in the ground state (i.e. in the first Bohr orbit) of a hydrogen atom to the speed of light in free space is of the order of

   A. 1 .

   B. 1/10 .

   C. 1/100 .

   D. 1/1000 .

J-56

6. The uncertainty product $\triangle x \triangle p_x$ for a one dimensional harmonic oscillator in the ground state is (where $\triangle x$ and $\triangle p_x$ are the uncertainties in position and momentum respectively)

A. $\hbar$.

B. $\hbar/2$.

C. $\hbar/4$.

D. 0.

7. The number of bound states supported by an attractive potential described by the Dirac-delta function in three dimensions is

A. 0.

B. 1.

C. 2.

D. ∞.

8. The degeneracy of the ground state of a three dimensional isotropic harmonic oscillator is

A. 0.

B. 1.

C. 2.

D. ∞.

9. The number of degrees of freedom for five particles constrained to move on the surface of a sphere is

A. 5.

B. 10.

C. 15.

D. 0.

10. A linear triatomic molecule such as carbon dioxide has

A. three vibrational modes, all at the same frequency.

B. four vibrational modes, two of which are at the same frequency.

C. four vibrational modes, each at a distinct frequency.

D. three vibrational modes, each at a distinct frequency.

11. From the Euler-Lagrange equation corresponding to the Lagrangian

$$L = \frac{m}{2}\left(\omega x + \frac{dx}{dt}\right)^2 + m\omega^2 x t \frac{dx}{dt}$$

it follows that L describes

A. a particle moving in a time independent harmonic potential.

B. a free particle.

C. a particle moving in a time dependent harmonic potential.

D. a particle moving in a linear potential.

12. If the Lagrangian describing the motion of a particle of mass m is $L = \frac{1}{2}m\dot{\mathbf{r}}^2 + \frac{k}{r}$, where k is a constant and $\mathbf{r}$ is the position vector of the particle, which of the following statements is <u>false</u> ?

A. The total energy of the particle is conserved.

B. The Laplace-Runge-Lenz vector associated with the particle is conserved.

C. The angular momentum of the particle is conserved.

D. The linear momentum of the particle is conserved.

13. If the electostatic field at a point $\mathbf{r}$ due to a system of static charges is $\mathbf{E}_1(\mathbf{r})$ and that due to another system is $\mathbf{E}_2(\mathbf{r})$ then the electrostatic energy density at $\mathbf{r}$ is given by

A. $\frac{\epsilon_0}{2}\left(\mathbf{E}_1(\mathbf{r}) \cdot \mathbf{E}_2(\mathbf{r})\right)$.

B. $\frac{\epsilon_0}{2}\left(\mathbf{E}_1^2(\mathbf{r}) - \mathbf{E}_2^2(\mathbf{r})\right)$.

C. $\frac{\epsilon_0}{2}\left(\mathbf{E}_1(\mathbf{r}) + \mathbf{E}_2(\mathbf{r})\right)^2$.

D. $\frac{\epsilon_0}{2}\left(\mathbf{E}_1^2(\mathbf{r}) + \mathbf{E}_2^2(\mathbf{r})\right)$.

14. If k_R and k_I denote the real and imaginary parts of the wave vector k of a plane electromagnetic wave propagating in a conducting medium then its wavelength (λ) inside the medium is given by

A. $\lambda = 2\pi/k_R$.

B. $\lambda = 2\pi/k_I$.

C. $\lambda = 2\pi/|k|$.

D. $\lambda = 2\pi/(k_R + ik_I)$.

J-56

15. The Poynting vector has the dimensions of

A. energy/(time × area).

B. energy/(time × volume).

C. (energy × time)/volume.

D. (energy × volume)/time.

16. The electric and the magnetic fields at a point due to a moving charge depend on

A. only the velocity and the retarded position of the moving charge.

B. only the acceleration and the retarded position of the moving charge .

C. only the velocity and the acceleration of the moving charge.

D. the velocity, the acceleration and the retarded position of the moving charge.

17. The specific heat at constant volume of an ideal Fermi gas, at temperatures $T \ll T_F$ (T_F is the Fermi temperature) varies with temperature as

A. T^2.

B. $1/T$.

C. T.

D. T^3.

18. If a thermodynamic system with fixed number of particles, volume and temperature is at equilibrium, then which of the following has a minimum?

A. Gibbs free energy.

B. Helmholtz free energy.

C. Heat capacity.

D. Entropy.

19. Rotations by which of the following angles is <u>not</u> a possible symmetry operation for a crystal lattice?

A. $\frac{2\pi}{3}$ radians.

B. $\frac{2\pi}{4}$ radians.

C. $\frac{2\pi}{6}$ radians.

D. $\frac{2\pi}{7}$ radians.

20. Two type I superconductors are separated by a thin insulating layer. To observe the Josephson effect in this system, the thickness of the insulating layer is of the order of

A. 20nm.

B. 1000 Å.

C. 1000nm.

D. 20 Å.

21. For a free electron of energy E and wave vector k inside a periodic potential, the group velocity v_g is given by

A. $\hbar \frac{dE}{dk}$.

B. $h \frac{dE}{dk}$.

C. $\frac{1}{\hbar} \frac{dE}{dk}$.

D. $\frac{1}{h} \frac{dE}{dk}$.

22. Which one of the following is not true for a junction field effect transistor (JFET) in comparison with a bipolar junction transistor (BJT).

A. The input impedance of a JFET is greater than that of BJT.

B. JFET has a higher current gain than a BJT.

C. JFET has a lower gain-bandwidth product than a BJT.

D. JFET is slower than a BJT.

23. An npn transistor with $\alpha = 0.98$ is operated in the common base configuration. If the emitter current is 5 mA and the reverse saturation current is 10 μA, then the base current is (α is the low voltage current amplification factor)

A. 90 μA.

B. 0.09 μA.

C. 5 mA.

D. 9.91 mA.

24. The speed of an electromagnetic wave propagating through a medium with dielectric constant $\epsilon = 10\epsilon_0$, where ϵ_0 is the dielectric constant in free space, is

A. $c/\sqrt{10}$.

B. $\sqrt{10}\,c$.

C. $c/10$.

D. $10c$.

25. Right and left circularly polarized monochromatic light beams with the angular frequency ω and wave vector k propagating in the positive x direction are coherently superposed. The resultant light beam is best described by

A. $\cos(kx - \omega t)$.

B. $\cos(kx - \omega t) + i\sin(kx - \omega t)$.

C. $\cos(kx - \omega t) - i\sin(kx - \omega t)$.

D. $\cos(kx - \omega t) + \sin(kx - \omega t)$.

J-56

SECTION - B

26. The value of the contour integral $\int_C dz\, z^2$ where C is a unit circle in the complex plane centered at the origin and traversed in an anti clockwise direction, is

  A. 1/3.

  B. $(1+i)/3$.

  C. $(1-i)/3$.

  D. 0.

27. The residue of the complex function $f(z) = \dfrac{1}{1+z^2}$ at $z = i$ is

  A. $-\dfrac{1}{2i}$.

  B. $-\dfrac{i}{2}$.

  C. 1.

  D. 0.

28. The Fourier transform $\tilde{f}(k) = \int_{-\infty}^{\infty} dx f(x) e^{-ikx}$ of the function $f(x) = e^{-\alpha x^2}$; $\alpha > 0$ is

  A. $\sqrt{\frac{\pi}{\alpha}} e^{-k^2/4\alpha}$.

  B. $\sqrt{\frac{\pi}{\alpha}} e^{-\alpha k^2}$.

  C. $\sqrt{\frac{\pi}{\alpha}} e^{ik/\sqrt{\alpha}}$.

  D. $\sqrt{\pi} e^{-k^2/4\alpha}$.

29. A 7×7 complex matrix has 3 zero eigenvalues. From this information alone one can conclude that its rank must be

  A. equal to 3.

  B. equal to 4.

  C. greater than or equal to 4 but less than 7.

  D. less than or equal to 3.

30. The power series $\sum_{n=0}^{\infty} \dfrac{2^{n+1}}{(2n+1)} (x - 3/2)^n$ converges at

  A. $x = 0$.

  B. $x = -1$.

  C. $x = 3$.

  D. $x = 5/4$.

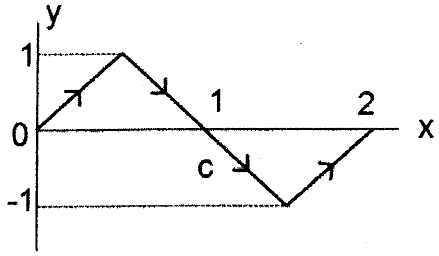
31. The value of integral $\int_C z\, dz$ along the open curve C in the complex z plane as shown below is

A. 0.

B. 1.

C. 2.

D. 4.



32. Let $\mathbf{r} = (1-t^2)/(1+t^2)\ \hat{\mathbf{i}} + 2t/(1+t^2)\ \hat{\mathbf{j}} + \hat{\mathbf{k}}$, where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors in the x, y and z directions respectively and $t \in \mathbb{R}$. The tip of the vector describes

A. an ellipse in the plane $z = 1$.

B. a parabola in the plane $z = 1$.

C. a circle in the plane $z = 1$.

D. a circle in the plane $z = 1$ except the point $x = -1$, $y = 0$, $z = 1$.

33. The substitution $x = \cos t$ reduces the equation $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ to

A. $\frac{d^2y}{dt^2} + \frac{\sin t}{\cos t}\frac{dy}{dt} = 0.$

B. $\frac{d^2y}{dt^2} = 0.$

C. $\frac{d^2y}{dt^2} + \frac{\cos t}{\sin t}\frac{dy}{dt} = 0.$

D. $\frac{d^2y}{dt^2} - \frac{\cos t}{\sin t}\frac{dy}{dt} = 0.$

34. If $[\hat{x}, \hat{p}_x] = i\hbar$ then $[\hat{x}^5, \hat{p}_x]$ equals

A. 0.

B. $4i\hbar\hat{x}$.

C. $5i\hbar\hat{p}_x$.

D. $5i\hbar\hat{x}^4$.

35. The eigenvalues of the Hamiltonian $\hat{H} = V_0\begin{pmatrix}1 & 0\\ 0 & 2\end{pmatrix} + \lambda V_0\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ upto the second order perturbation for $V_0 > 0$, $1 \gg \lambda > 0$, is

A. $V_0(1+\lambda^2)$ and $V_0(2+\lambda^2)$.

B. $V_0(1-\lambda^2)$ and $V_0(2-\lambda^2)$.

C. $V_0(1-\lambda^2)$ and $V_0(2+\lambda^2)$.

D. $V_0(1+\lambda^2)$ and $V_0(2-\lambda^2)$.

36. For two spin-1/2 particles the difference between the expectation values of the operator $\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$ in the triplet and the singlet states (where $\hat{\mathbf{S}}_1, \hat{\mathbf{S}}_2$ are the corresponding spin operators) is

A. $\hbar^2$.

B. $\hbar^2/4$.

C. $3\hbar^2/4$.

D. $\hbar^2/2$.

37. In relativistic quantum mechanics, which of the following is the correct representation of the spin operators ($\hat{S}_j$) for a spin-1/2 particle (Here $\{\sigma_j\}$ denote the Pauli spin-matrices and I the 2×2 identity matrix)

A. $S_j = \frac{\hbar}{2}\begin{pmatrix} \sigma_j & 0 \\ 0 & I \end{pmatrix}$.

B. $S_j = \frac{\hbar}{2}\begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix}$.

C. $S_j = \frac{\hbar}{2}\begin{pmatrix} I & 0 \\ 0 & \sigma_j \end{pmatrix}$.

D. $S_j = \frac{\hbar}{4}\begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix}$.

38. Typical spin-orbit coupling energy ($\triangle E^{L-S}$) for a Hydrogen atom in the ground state, in terms of relativistic energy (mc^2) and fine-structure constant (α), varies as

A. $mc^2\alpha$.

B. $mc^2\alpha^2$.

C. $mc^2\alpha^3$.

D. $mc^2\alpha^4$.

39. A relativistic particle of rest mass m_0 is moving with a speed v. The value of v at which its kinetic energy is equal to its rest mass energy (m_0c^2) is

A. $v = c/2$.

B. $v = c/4$.

C. $v = c/\sqrt{2}$.

D. $v = c(\sqrt{2} - 1)$.

40. A particle of mass m moving with velocity $\mathbf{v_1}$ leaves a region in which its potential energy is a constant U_1 and enters another in which its potential energy is a different constant U_2. If θ_1 and θ_2 are the angles between the normal to the plane and the velocities $\mathbf{v_1}$ and $\mathbf{v_2}$ of the particle before and after passing the plane of separation of the two regions, then

A. $\frac{v_2}{v_1} = \sqrt{[1 - \frac{2}{mv_1^2}(U_1 - U_2)]} \cdot \left(\frac{\cos\theta_1}{\cos\theta_2}\right)$.

B. $\frac{v_2}{v_1} = \sqrt{[1 + \frac{2}{mv_1^2}(U_1 - U_2)]}$.

C. $\frac{v_2}{v_1} = \sqrt{[1 + \frac{2}{mv_1^2}(U_1 + U_2)]}$.

D. $\frac{v_2}{v_1} = \sqrt{[1 - \frac{2}{mv_1^2}(U_1 + U_2)]}$.

41. The constraint conditions for a bead moving on a smooth circular wire of radius r centered at the origin and lying in the yz-plane, are

A. $x = 0, \quad y^2 + z^2 = r^2$.

B. $z = 0, \quad x^2 + y^2 = r^2$.

C. $y = 0, \quad x^2 + z^2 = r^2$.

D. $y = z = 0, \quad x = r$.

42. If $\hat{L}_x, \hat{L}_y, \hat{L}_z$ denote the components of angular momentum operator and $\hat{p}_x, \hat{p}_y, \hat{p}_z$ those of the linear momentum operator then $[L_x, [L_y, P_z]] + [L_y, [P_z, L_x]]$ is equal to

A. 0.

B. $\hat{L}_z$.

C. $\hat{p}_z$.

D. $\hat{L}_x + \hat{L}_y$.

43. A solid spherical object (of radius r and mass m) is rolling down (without slip) along an inclined plane which makes an angle θ with the horizontal line. Acceleration of the particle (in terms of the acceleration due to the gravity g) is

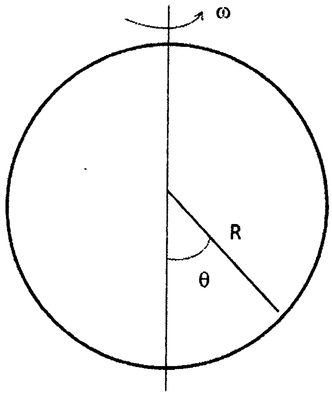
A. $\frac{2}{7} g \sin\theta$.

B. $\frac{2}{5} g \sin\theta$.

C. $\frac{5}{7} g \sin\theta$.

D. $g \sin\theta$.

44. A bead of mass m can slide without friction on a vertical hoop of radius R. The hoop rotates at a constant angular speed ω about a vertical axis passing through the centre of the hoop as shown:



The Lagrangian for the system, including effects of gravity, can be expressed in terms of the generalized co-ordinate θ as

A. $\frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\omega^2\sin^2\theta + mgR\cos\theta$.

B. $\frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\omega^2 + mgR\cos\theta$.

C. $\frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\omega^2\sin^2\theta + mgR$.

D. $\frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\omega^2\sin^2\theta - mgR\cos\theta$.

45. For a system in three dimensions described by the Lagrangian $L = m\left(\frac{d\mathbf{r}}{dt}\right)^2 + a(x^2 + y^2) + bz^2$, where m, a, b are constants, which of the following statements is true

A. The total energy, the linear momentum $\mathbf{p}$ and angular momentum $\mathbf{L}$ are conserved.

B. The total energy, linear momentum $\mathbf{p}$ and the x and y components L_x, L_y of the angular momentum are conserved.

C. The total energy and the z-component L_z of the angular momentum are conserved.

D. The total energy, x and y components p_x, p_y of the linear momentum and the angular momentum $\mathbf{L}$ are conserved.

46. The Lagrangian for a system described by the Hamiltonian $H = \frac{p^2}{2m}e^{ax} + V(x)$ is

A. $\frac{3m}{2}\dot{x}^2e^{ax} - V(x)$.

B. $\frac{3m}{2}\dot{x}^2e^{ax} + V(x)$.

C. $\frac{m}{2}\dot{x}^2e^{-ax} - V(x)$.

D. $\frac{m}{2}\dot{x}^2e^{-ax} + V(x)$.

47. The electrostatic potential for a system is given by $\phi(r) = \alpha \frac{e^{-\beta r}}{r}$, where α and β are constants. The electric field $\mathbf{E}(\mathbf{r})$ at the point $\mathbf{r}$ is

A. $-\frac{\alpha\beta e^{-\beta r}}{r}\hat{\mathbf{r}}$.

B. $\alpha e^{-\beta r}\beta\frac{\hat{\mathbf{r}}}{r^2}$.

C. $\frac{\alpha e^{-\beta r}(1-\beta)\hat{\mathbf{r}}}{r^2}$.

D. $\frac{\alpha e^{-\beta r}(1+\beta)\hat{\mathbf{r}}}{r^2}$.

48. A long cylinder carries a charge density, $\rho = \alpha r$, where α is a constant and r denotes the distance from its axis. The magnitude of the electric field at a point inside the cylinder at a distance r from the axis of the cylinder is

A. $\frac{1}{\epsilon_0}\pi\alpha r^2$.

B. $\frac{1}{3\epsilon_0}\pi\alpha r^2$.

C. $\frac{2}{3\epsilon_0}\pi\alpha r$.

D. $\frac{1}{4\pi\epsilon_0}\alpha r^3$.

49. The magnitude of the magnetic field, due to an infinite uniform surface current $\mathbf{K} = K\hat{i}$ covering the xy plane at point located at a distance d from the xy plane, is

A. $\mu_0 K$.

B. 0.

C. $\frac{\mu_0 K}{2d}$.

D. $\frac{\mu_0}{2}K$.

50. If the electric field $\mathbf{E}(z,t)$ of a plane wave propagating in a conducting medium along the positive z-direction is $\mathbf{E}(z,t) = E_0[e^{i(\kappa z-\omega t)} + e^{-i(\kappa^* z-\omega t)}]\,\hat{\mathbf{i}}$, where $\kappa = \kappa_R + i\kappa_I$, the corresponding magnetic field $\mathbf{B}(z,t)$ is given by

A. $\mathbf{B}(z,t) = \frac{E_0|\kappa|}{\omega}[e^{i(\kappa z-\omega t)} + e^{-i(\kappa^* z-\omega t)}]\,\hat{\mathbf{j}}$.

B. $\mathbf{B}(z,t) = \frac{E_0\kappa_R}{\omega}[e^{i(\kappa z-\omega t)} + e^{-i(\kappa^* z-\omega t)}]\,\hat{\mathbf{j}}$.

C. $\mathbf{B}(z,t) = \frac{E_0}{\omega}[\kappa e^{i(\kappa z-\omega t)} + \kappa^* e^{-i(\kappa^* z-\omega t)}]\,\hat{\mathbf{j}}$.

D. $\mathbf{B}(z,t) = \frac{E_0 i\kappa_I}{\omega}[e^{i(\kappa z-\omega t)} - e^{-i(\kappa^* z-\omega t)}]\,\hat{\mathbf{j}}$.

51. The Lagrangian which appropriately describes the motion of a particle of charge q and mass m in an external electromagnetic field specified by the scalar potential ϕ and the vector potential $\mathbf{A}$ is

A. $L = \frac{1}{2}m\dot{\mathbf{r}}^2 - q(\phi - \mathbf{A}\cdot\dot{\mathbf{r}})$.
B. $L = \frac{1}{2}m\dot{\mathbf{r}}^2 - q(\phi + \mathbf{A}\cdot\dot{\mathbf{r}})$.
C. $L = \frac{1}{2}m\dot{\mathbf{r}}^2 + q(\phi - \mathbf{A}\cdot\dot{\mathbf{r}})$.
D. $L = \frac{1}{2}m\dot{\mathbf{r}}^2 + q(\phi + \mathbf{A}\cdot\dot{\mathbf{r}})$.

52. The acceleration dependent part of the electric and magnetic fields at a point in the radiation zone due to a moving charge depends on its retarded distance R as (assuming its velocity to be much less than that of light)

A. R.
B. R^2.
C. R^{-2}.
D. R^{-1}.

53. If a quantity of heat Q added to an ideal monoatomic gas, at constant volume, results in a temperature change of ΔT, then the heat input required to produce the same temperature change at constant pressure is

A. $\frac{3}{2}Q$.
B. $\frac{1}{2}Q$.
C. $\frac{5}{3}Q$.
D. $\frac{2}{3}Q$.

54. The value of the translational partition function for Ar gas at a temperature T_1 is Z_1. If the mass of the gas is m and occupies a volume V then its value at a temperature $T_2 = T_1/4$, assuming m and V to be constant, is

A. $4Z_1$.
B. $\frac{1}{4}Z_1$.
C. $\frac{1}{2}Z_1$.
D. $\frac{1}{3}Z_1$.

J-56

55. The equation of state of a 3-dimensional ideal Fermi gas is given by the relation

$$p = \frac{2}{5}\epsilon_F \tilde{n}\left[1 + \frac{5C}{12}\pi^2\left(\frac{k_BT}{\epsilon_F}\right)^2\right]$$

where p is the pressure, T temperature, $\tilde{n}$ is the number density of particles, ϵ_F is the Fermi energy, and C is a constant. Then the specific heat per particle at constant volume c_v is given by

A. $\frac{3C}{2}k_B$.

B. $\frac{3k_BC}{2}\left(\frac{k_BT}{\epsilon_F}\right)$.

C. $k_B\frac{\pi^2}{2}\left(\frac{k_BT}{\epsilon_F}\right)$.

D. $k_B\frac{\pi^2 C}{2}\left(\frac{k_BT}{\epsilon_F}\right)$.

56. The temperature at which the thermal de Broglie wavelength becomes comparable to the average separation of particle of number density $10^{21}/\mathrm{m}^3$ of a 3-dimensional ideal gas of ^{23}Na is

A. 1 μK.

B. 0.1 μK.

C. 10 μK.

D. 100 μK.

57. Consider the particle decay $A \to B + C$, where B and C are produced in a relative P state of angular momentum. If the intrinsic parity of B is $+1$ and that of C is -1, then the intrinsic parity of A is

A. -1.

B. $+1$.

C. 0.

D. ± 1.

58. A particle of rest mass m and total energy E collides with a stationary particle of equal mass. If $E \gg m$, the energy available in the centre of momentum system for particle creation is proportional to

A. E.

B. $\sqrt{E}$..

C. E^2.

D. 2E.

J-56

59. The correct pair of Miller indices for Bragg reflections from a BCC and a FCC crystal respectively are

A. (1 0 0); (1 1 0).
B. (1 1 0); (1 1 1).
C. (3 1 0); (1 0 0).
D. (1 1 1); (2 0 0).

60. Considering atoms as hard and uniform spheres that are packed closely together in a face centered cubic structure, the fraction of the volume occupied by the atoms is

A. 52%.
B. 65%.
C. 74%.
D. 88%.

61. The value of Landé g factor for a magnetic ion in the ${}^6S_{5/2}$ state is

A. 1.
B. 2.
C. 3.
D. 17/2.

62. The ratio of the interplanar spacing of (1 1 0) and (1 1 1) planes in a simple cubic crystal is

A. $1 : \sqrt{2}$.
B. $\sqrt{2} : \sqrt{3}$.
C. $\sqrt{3} : \sqrt{2}$.
D. $\sqrt{2} : 1$.

63. For a crystal, in which the speed of sound is $c/10000$ (where c is speed of light) and the number density of atoms is $10^{27}/\text{m}^3$, the Debye temperature is approximately

A. 2 K.
B. 20 K.
C. 200 K.
D. 500 K.

J-56

64. If a superconducting sample of lead has a critical magnetic field of 6.5×10^4 ampere/meter and a critical temperature of 7 K in zero magnetic field then the critical magnetic field at 3.5 K is

A. 0.9×10^4 A/m.
B. 4.9×10^4 A/m.
C. 5.7×10^4 A/m.
D. 6.5×10^4 A/m.

65. The Debye temperatures of two isomorphous ionic solids A and B are 300 K and 250 K respectively. If the lattice heat capacity of A at 5 K is 0.050 J/mol.K, then the value of heat capacity of B at 2.5 K is

A. $0.086 J/(\text{mol} \cdot K)$.
B. $0.050 J/(\text{mol} \cdot K)$.
C. $0.011 J/(\text{mol} \cdot K)$.
D. $0.001 J/(\text{mol} \cdot K)$.

66. If the electrons in a semiconductor with mobility $7 \times 10^{-3} \text{m}^2/\text{Vs}$ are accelerated by an applied electric field strength of 1V/cm then the drift velocity is

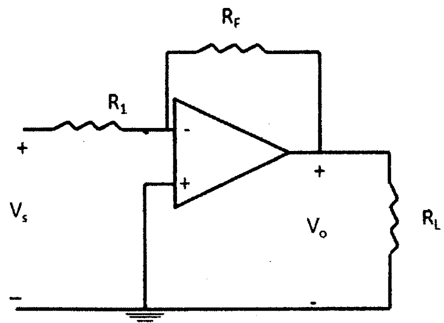
A. 70m/s.
B. 7×10^{-3}m/s.
C. 7×10^{-5}m/s.
D. 0.7m/s.

67. The reverse saturation current (I_{CBO}) of the collector base junction of a BJT is found to be 10 nA at low collector voltages. If the low voltage current amplification factor $\alpha =$ 0.98, then the change in collector current (I_{CEO}), with its base open, when the collector voltage is increased to cause an increase of 1.0% in α is

A. 480 nA.
B. 480 mA.
C. 980 nA.
D. 500 μA.

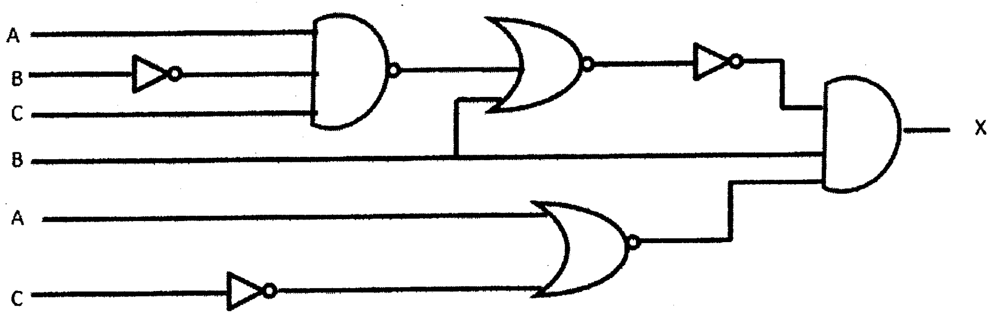
68. In the op-amp circuit given below if R_L is increased three times then the output voltage V_0

A. does not increase.
B. becomes three times.
C. becomes one third.
D. becomes equal to $(R_L/R_F) \cdot V_s$.

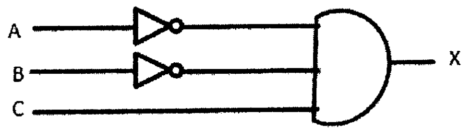


J-56

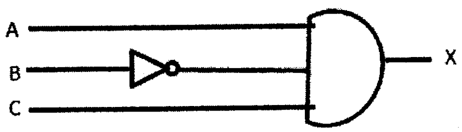
69. The equivalent circuit of the logic circuit given below is



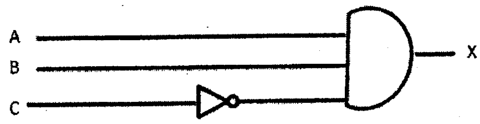
A.



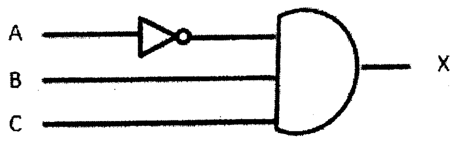
B.



C.

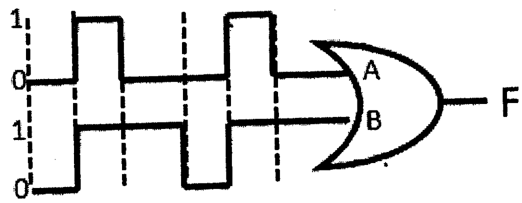


D.

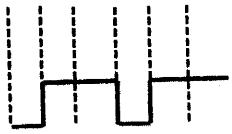


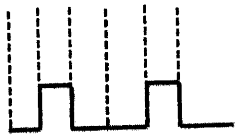
I-56

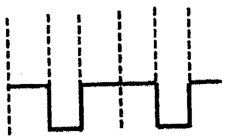
70. The timing diagrams for a two input OR gate are given below.

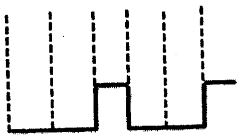


The timing diagram corresponding to the output, F is

A.


B.


C.


D.


71. A He-Ne laser beam of intensity 1 mW, radius r =0.5 mm is positioned at a distance of d = 2 m in front of the eye and the entire beam enters the pupil. The intensity of the beam at the retina of your eye when you look directly into the beam is

A. 196 kW/m^2.
B. 392 kW/m^2.
C. 196 W/m^2.
D. 196 MW/m^2.

72. For a plane wave propagating with a phase velocity $v_p = a\sqrt{\lambda}$, where λ is the wavelength, the group velocity v_g is

A. $v_p + \frac{a}{2}\sqrt{\lambda}$.
B. $v_p - a\sqrt{\lambda}$.
C. $\frac{a}{2}\sqrt{\lambda}$.
D. $v_p - \frac{a}{2}\sqrt{\lambda}$.

J-56

73. Symmetric diffraction rings are formed on a screen placed at a distance of 1.8 m from a circular aperture of 0.5 mm diameter, illuminated by a monochromatic light of wavelength=680 nm. The angle made by the second dark ring observed on the screen with reference to the symmetry axis of the setup is

A. 2.72 mrad.

B. 1.36 mrad.

C. 5.44 mrad.

D. 1.51 mrad.

74. Consider the zeroth order bright fringe formed in the Young's double slit experiment illuminated by a white light source. The separation between the slits is b and is kept at a distance of z from the screen. If one of the slits is covered by a transparent plate of thickness t and refractive index n, the lateral displacement of the bright fringe is

A. $(n-1)tz/b$.

B. ntz/b.

C. $(n+1)tz/b$.

D. $(n-1)tzb$.

75. The relationship between four physical quantities T, k, a and C is given by the equation

$$T = \frac{\pi}{a}\left(\frac{k+C}{2}\right)^{1/2}$$

If T and k are the measured quantities then which one of the following statements is correct?

A. The x-intercept of the graph between T^2 on the y-axis and k on the x-axis leads to the value of a.

B. The slope of the graph between T on the y-axis and k^2 on the x-axis leads to the value of a.

C. The slope of the graph between T on the y-axis and C on the x-axis leads to the value of a.

D. The slope of the graph between T^2 on the y-axis and k on the x-axis leads to the value of a.