H-06

Entrance Examination : M.Sc. Mathematics, 2015

Hall Ticket Number

Time : 2 hours
Max. Marks. 100

Part A : 25 marks
Part B : 75 marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR answer sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination.
5. The question paper can be taken by the candidate at the end of the examination.
6. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
7. Calculators are not allowed.
8. There are a total of 50 questions in Part A and Part B together.
9. **There is a negative marking in Part A. Each correct answer carries 1 mark and each wrong answer carries −0.33 mark. Each question in Part A has only one correct option.**
10. **There is no negative marking in Part B. Each correct answer carries 3 marks. In Part B some questions have more than one correct option. All the correct options have to be marked in OMR sheet other wise zero marks will be credited.**
11. The appropriate answer(s) should be colored with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
12. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers and $\mathbb{N}$ the set of natural numbers.
13. This book contains 9 pages including this page and excluding page for the rough work, Please check that your paper has all the pages.

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Part-A

1. Let A be an $m \times n$ matrix and B be an $n \times m$ matrix. Then which of the following statements is true?

   (A) $\text{rank}(AB) > \min(\text{rank}(A), \text{rank}(B))$.

   (B) $\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$.

   (C) $\text{rank}(AB) \leq \max(\text{rank}(A), \text{rank}(B)) - \min(\text{rank}(A), \text{rank}(B))$.

   (D) $\text{rank}(AB) > \max(\text{rank}(A), \text{rank}(B)) - \min(\text{rank}(A), \text{rank}(B))$.

2. Let A be an $n \times n$ nonzero matrix where A is not an identity matrix. If $A^2 = A$, then the eigenvalues of A are given by

   (A) 1 and -1. (B) 0 and 1. (C) -1 and 0. (D) 0 and n.

3. Let A be a 7×5 matrix over $\mathbb{R}$ having at least 5 linearly independent rows. Then the dimension of the null space of A is

   (A) 0. (B) 1. (C) 2. (D) at least 2.

4. The dimension of the vector subspace W of $M_2(\mathbb{C})$ given by
   $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{C}, a + b = c,\ b + c = d,\ c + a = d \right\}$ is equal to

   (A) 4. (B) 3. (C) 2. (D) 1.

5. If $|a - b| = |c - d|$, then

   (A) $a = b + c - d$. (B) $a = b - c + d$.

   (C) $a = b + c - d$ and $a = b - c + d$. (D) $a = b + c - d$ or $a = b - c + d$.

6. The set of all real numbers x for which there is some positive real number y such that $x < y$ is equal to

   (A) $\mathbb{R}$. (B) the set of all negative real numbers.

   (C) $\{0\}$. (D) the empty set.

7. Let $\hat{n}$ be the unit outward normal to the sphere of radius α in $\mathbb{R}^3$. Then the value of the integral $\int \vec{r} \cdot \hat{n}\, dS$ evaluated on the sphere is equal to

   (A) $\frac{4}{3}\pi\alpha^3$. (B) $4\pi\alpha^2$. (C) $\frac{4}{3}\pi\alpha^2$. (D) $4\pi\alpha^3$.

8. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = \sqrt{x^2 + y^2 + z^2}$ and $n \in \mathbb{N}$ then ∇r^n is equal to

   (A) $nr^{n-1}\vec{r}$. (B) $(n-1)r^{n-2}\vec{r}$. (C) $nr^{n-2}\vec{r}$. (D) $(n-1)r^n\vec{r}$.

9. The value of the integral $\int_C \left(\frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy \right)$ where C is the circle with radius α centered at the origin is equal to

(A) 0. (B) $\frac{\pi}{2}$. (C) 2π. (D) $2\pi\alpha$.

10. The volume of the cube whose two faces lie on the planes $6x - 3y + 2z + 1 = 0$ and $6x - 3y + 2z + 4 = 0$ is equal to

(A) 27. (B) $\frac{27}{343}$. (C) $\frac{3}{7}$. (D) 10.

11. The number of common tangent planes to the spheres $(x+2)^2 + y^2 + z^2 = 1$, $(x-2)^2 + y^2 + z^2 = 1$ passing through the origin is equal to

(A) 0. (B) 1. (C) 2. (D) none of these.

12. Let c be an arbitrary nonzero constant. Then the orthogonal family of curves to the family $y(1 - cx) = 1 + cx$ is

(A) $3y - y^3 + 3x^2 =$ constant. (B) $3y + y^3 - 3x^2 =$ constant.
(C) $3y - y^3 - 3x^2 =$ constant. (D) $3y + y^3 + 3x^2 =$ constant.

13. Consider the following two statements.

S_1: If (a_n) is any real sequence, then $\left(\frac{a_n}{1 + |a_n|} \right)$ has a convergent subsequence.

S_2: If every subsequence of (a_n) has a convergent subsequence, then (a_n) is bounded.

Which of the following statements is true?

(A) Both S_1 and S_2 are true. (B) Both S_1 and S_2 are false.
(C) S_1 is false but S_2 is true. (D) S_1 is true but S_2 is false.

14. The largest interval I such that the series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ converges whenever $x \in I$ is equal to

(A) $[-1, 1]$. (B) $[-1, 1)$. (C) $(-1, 1]$. (D) $(-1, 1)$.

15. Let $\sum a_n$ be a convergent series. Let $b_n = a_{n+1} - a_n$ for all $n \in \mathbb{N}$. Then

(A) $\sum b_n$ should also be convergent and $(b_n) \to 0$ as $n \to \infty$.
(B) $\sum b_n$ need not be convergent but $(b_n) \to 0$ as $n \to \infty$.
(C) $\sum b_n$ is convergent but (b_n) need not tend to zero as $n \to \infty$.
(D) none of the above statements is true.

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16. Consider the real sequences (a_n) and (b_n) such that $\sum a_n b_n$ converges. Which of the following statements is true?

(A) If $\sum a_n$ converges, then (b_n) is bounded.
(B) If $\sum b_n$ converges, then (a_n) is bounded.
(C) If (a_n) is bounded, then (b_n) converges.
(D) If (a_n) is unbounded, then (b_n) bounded.

17. If $f : \mathbb{R} \to \mathbb{R}$ and $\lim_{h \to 0}(f(x+h) - f(x-h)) = 0$ for all $x \in \mathbb{R}$, then

(A) f need not be continuous.
(B) f is continuous but not differentiable.
(C) f is differentiable but f' need not be continuous.
(D) f is differentiable and f' is continuous.

18. If $f : [0,1] \to \mathbb{R}$ is continuous and $f(1) < f(0)$, then

(A) $f([0,1]) \subseteq [f(1), f(0)]$. (B) $f([0,1]) \supseteq [f(1), f(0)]$.
(C) $f([0,1]) = [f(1), f(0)]$. (D) $f([0,1])$ need not be a closed interval.

19. Consider $f : [-1, 2] \to \mathbb{R}$ defined by $f(x) = \begin{cases} -x, & \text{if } -1 \le x \le 0 \\ 2x^3 - 4x^2 + 2x, & \text{if } 0 < x \le 2. \end{cases}$
Then the maximum value of $f(x)$ is equal to

(A) 0. (B) 2. (C) 4. (D) 10.

20. The function e^x from $\mathbb{R}$ to $\mathbb{R}$ is

(A) both one-one and onto. (B) one-one but not onto.
(C) onto but not one-one. (D) neither one-one nor onto.

21. The number of elements of order 6 in a cyclic group of order 36 is equal to

(A) 2. (B) 3. (C) 4. (D) 6.

22. Consider the following two statements.

S_1: There cannot exist an infinite group in which every element has a finite order.
S_2: In a group G if $a \in G$, $a^7 = e$ and $a^9 = e$, then $a = e$.

Which of the following statements is true?

(A) Both S_1 and S_2 are true.
(B) Both S_1 and S_2 are false.
(C) S_1 is false but S_2 is true.
(D) S_1 is true but S_2 is false.

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23. Let R be a commutative ring with unity and $1 \neq 0$. Let a be a nilpotent element, x be a unit. Then

(A) $1 + a$ is not a unit.
(B) $a - x$ is a nilpotent element.
(C) $x + a$ is a unit.
(D) none of the above statements is true.

24. Let R be a commutative ring with unity. Consider the following two statements.

S_1: If for any $a \in R$, $a^2 = 0$ implies $a = 0$ then R does not have nonzero nilpotent elements.

S_2: If A and B are two ideals of R with $A + B = R$ then $A \cap B = AB$.

Then which of the following statements is true?

(A) Both S_1 and S_2 are true.
(B) Both S_1 and S_2 are false.
(C) S_1 is false but S_2 is true.
(D) S_1 is true but S_2 is false.

25. In how many ways can one place 8 identical balls in 3 different boxes so that no box is empty?

(A) 8. (B) 28. (C) 36. (D) 21.

Part-B

26. The projection of the point $(11, -1, 6)$ onto the plane $3x + 2y - 7z - 51 = 0$ is equal to

(A) $(14, 1, -1)$. (B) $(4, 2, -5)$. (C) $(18, 2, 1)$. (D) none of these.

27. The projection of the straight line $x - y - z = 0$ and $2x + 3y + z = 5$ onto the $yz-$ plane is

(A) $5y = -3z + 5$ and $x = 0$.
(B) $y = 3z + 5$ and $x = 0$.
(C) $y = z + 5$ and $x = 0$.
(D) $y = -z + 5$ and $x = 0$.

28. The number of spheres of radii $\sqrt{2}$ such that the area of each circle of intersection with the three coordinate planes is π is equal to

(A) 1. (B) 3. (C) 4. (D) 8.

29. If all blind horses are white then it follows that

(A) no blind horse is black.
(B) no brown horse is blind.
(C) all white horses are blind.
(D) all horses are blind and white.

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30. The set of all real roots of the polynomial $P(x) = x^4 - x$ is

(A) $\{0, 1\}$.
(B) the set of roots of $(x^2 - x)$.
(C) a set having four elements.
(D) an infinite set.

31. Let $f, g : \mathbb{R} \to \mathbb{R}$ be polynomials. Then which of the following are **false**?

(A) If $f(x) = g(x)$ for all $x \in [0, 1]$ then $f = g$.
(B) If $f\left(\frac{1}{n}\right) = g\left(\frac{1}{n}\right)$ for all $n \in \mathbb{N}$ then $f = g$.
(C) If $f(x) \leq g(x)$ for all $x \in \mathbb{R}$ then degree$(f) \leq$ degree(g).
(D) If $\{x \in \mathbb{R} : f(x) = 0\} = \{x \in \mathbb{R} : g(x) = 0\}$ then $f = g$.

32. If the graph of the function $y = f(x)$ is symmetrical about the line $x = a$, then

(A) $f(x) = f(-x)$.
(B) $f(x + a) = f(-x - a)$.
(C) $f(x + a) = f(a - x)$.
(D) $f(2a - x) = f(x)$.

33. Consider the following two statements.

S_1: There exists a linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^2$ such that T is onto and $\text{Ker}(T) = \{(x_1, x_2, x_3, x_4, x_5) : x_1 + x_2 + x_3 = 0\}$.

S_2: For every linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ there exists $\mu \in \mathbb{R}$ such that $T - \mu I$ is invertible.

Which of the following statements are true?

(A) Both S_1 and S_2 are true.
(B) Both S_1 and S_2 are false.
(C) S_1 is false but S_2 is true.
(D) S_1 is true but S_2 is false.

34. The set $S = \{-1, 1\}$ is the set of eigenvalues of the square matrix A, if

(A) $A \pm I \neq 0$, A is a real, orthogonal and symmteric matrix.
(B) $A \pm I \neq 0$, A is a symmteric matrix.
(C) $A \pm I \neq 0$, $A^2 = I$.
(D) $A \pm I \neq 0$, A is a Hermitian matrix.

35. If $A \neq 0$ is a 2×2 real matrix and suppose $A^2\vec{v} = -\vec{v}$ for all vectors $\vec{v} \in \mathbb{R}^2$, then

(A) -1 is an eigenvalue of A.
(B) the characteristic polynomial of A is $\lambda^2 + 1$.
(C) the map from $\mathbb{R}^2 \to \mathbb{R}^2$ given by $\vec{v} \mapsto A\vec{v}$ is surjective.
(D) det A $=1$.

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36. Consider a linear system of equations $A\vec{x} = \vec{b}$ where A is a 3×3 matrix and $\vec{b} \neq 0$. Suppose the rank of the matrix of coefficients $A = (a_{ij})$ is equal to 2 then

(A) there definitely exists a solution to the system of equations.

(B) there exists a non-zero column vector $\vec{v}$ in $\mathbb{R}^3$ such that $A\vec{v} = \vec{0}$.

(C) if there exists a solution to the system of equations $A\vec{x} = \vec{b}$ then at least one equation is a linear combination of the other two equations.

(D) $\det A = 0$.

37. Which of the following sets are closed and bounded?

(A) $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 3\}$.
(B) $\{(x,y) \in \mathbb{R}^2 : x + y = 3\}$.
(C) $\{(x,y) \in \mathbb{R}^2 : |x| + |y| \leq 3\}$.
(D) $\{(x,y) \in \mathbb{R}^2 : \max\{|x|, |y|\} \leq 3\}$.

38. Let $\ell \in \mathbb{R}$, and (a_n) be a real sequence. Then which of the following conditions is equivalent to '$(a_n) \to \ell$ as $n \to \infty$'?

(A) $\forall \epsilon > 0, \exists n_0 \in \mathbb{N}$ such that $|a_n - \ell| < 2\epsilon$ whenever $n \geq n_0$.

(B) $\forall \epsilon > 0, \exists n_0 \in \mathbb{N}$ such that $|a_n - \ell| < \epsilon$ whenever $n \geq 2n_0$.

(C) $\forall \epsilon > 0, \exists n_0 \in 3\mathbb{N}$ such that $|a_n - a_m| < 2\epsilon$ whenever $m, n \geq n_0$.

(D) $\forall \epsilon > 0, \exists n_0 \in \mathbb{N}$ such that $|a_n - a_m| < 2\epsilon$ whenever $m, n \geq n_0$.

39. Which of the following series converge?

(A) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\log n}{n^{1+2\epsilon}}\right)$.
(B) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{(\log n)^2}{n^{1+2\epsilon}}\right)$.
(C) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{n^3+n}\right)$.
(D) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$.

40. Let $f(x) = \begin{cases} x^{3/2}(1-x)^{5/4}, & x \in (0,1), \\ 0, & x \in \mathbb{R} \setminus (0,1). \end{cases}$ Then

(A) f is discontinuous at 0 and 1.

(B) f is continuous but not differentiable at 0 and 1.

(C) f is differentiable at 0 and 1 but f' is not continuous at 0 and 1.

(D) none of the above.

41. The value of the integral $\displaystyle\int_0^2 \left(x - [x^2]\right) dx$ is equal to

(A) $\sqrt{2}+\sqrt{3}+3$.
(B) $\sqrt{2}+\sqrt{3}-3$.
(C) $\sqrt{2}-\sqrt{3}+3$.
(D) $\sqrt{2}-\sqrt{3}-3$.

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42. Let f and g be real valued functions on $[0,1]$ which are Riemann integrable. Let $f(x) \leq g(x)$ for all $x \in [0,1]$ and $f\left(\frac{1}{2}\right) < g\left(\frac{1}{2}\right)$. The inequality $\int f dx < \int g dx$ holds if

(A) f and g are continuous in $[0,1]$.
(B) f is continuous.
(C) g is continuous.
(D) f and g are continuous in a neighbourhood containing $\frac{1}{2}$.

43. The general solution of $y''' - 4y'' + y' = 0$ is

(A) $c_1 \sinh^2 x + c_2 \cosh^2 x + c_3$. (B) $c_1 \sinh 2x + c_2 \cosh 2x + c_3$.
(C) $c_1 \sin 2x + c_2 \cos 2x + c_3$. (D) $c_1 e^{2x} + c_2 e^{-2x} + c_3$.

44. Which of the following are solutions of the differential equation $yy'' - (y')^2 + 1 = 0$?

(A) x.
(B) $\sin(x + c)$ where c is an arbitrary constant.
(C) $\sinh(x + c)$ where c is an arbitrary constant.
(D) none of the above.

45. Which of the following statements are true?

(A) In a cyclic group of order n, if m divides n, then there exists a unique subgroup of order m.
(B) A cyclic group of order n will have $(n-1)$ elements of order n.
(C) In a cyclic group of order 24 there is a unique element of order 2.
(D) In the group $(\mathbb{Z}_{12}, +)$ of integers modulo 12 the order of $\overline{5}$ is 12.

46. Let G be a finite group with no nontrivial proper subgroups. Then which of the following statements are true?

(A) G is cyclic. (B) G is abelian.
(C) G is of prime order. (D) G is non-abelian.

47. The equation $5X = 7 (\mathrm{mod}\ 12)$ has

(A) a unique solution in $\mathbb{Z}$.
(B) a unique solution in the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.
(C) a unique solution in the set $\{n, n+1, n+2, n+3, n+4, n+5, n+6, n+7, n+8, n+9, n+10, n+11\}$.
(D) no solution in $\mathbb{Z}$.

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48. Which of the following maps are ring homomorphisms?

(A) $f : \mathbb{Z}_4 \to \mathbb{Z}_{10}$, $f(x) = 5x$.

(B) $f : \mathbb{Z}_5 \to \mathbb{Z}_{10}$, $f(x) = 5x$.

(C) $f : \mathbb{Z}_4 \to \mathbb{Z}_{12}$, $f(x) = 3x$.

(D) $f : \mathbb{Z}_4 \to R$, $f(x) = xe$ where R is a ring with unity e.

49. Let R be a finite commutative ring with no zero divisors then

(A) R is a field.

(B) R has a unity.

(C) characteristic of R is a prime number.

(D) none of the above.

50. Each question in a text has 4 options of which only one is correct. Ashok does not know which of the options are correct or wrong in 3 questions. He decides to select randomly the options for these 3 questions independently. The probability that he will choose at least 2 correctly is

(A) more than 0.25.

(B) in the interval $(0.2, 0.25]$.

(C) in the interval $(1/6, 0.2]$.

(D) less than $1/6$.