ENTRANCE EXAMINATIONS – 2022

Ph.D. Statistics

Hall Ticket Number	

Time : 2 hours

Max. Marks. 70

Part A : 35 marks

Part B : 35 marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR answer sheet.
3. Hand over the OMR answer sheet after the examination to the Invigilator.
4. No additional sheets will be provided.Rough work can be done in the question paper itself/ space provided at the end of the booklet.
5. Calculators are not allowed.
6. There are a total of 70 questions in **PART - A** and **PART - B** together of **one** mark each.
7. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
8. This book contains **14** pages including this page and excluding pages for rough work. Please check that your paper has all the pages.
9. Given below are the meanings of some symbols that may have appeared in the question paper:
   $\mathbb{R}$-The set of all real numbers,$E(X)$-Expected value of the random variable X,
   $V(X)$-Variance of the random variable X, $Cov.(X,Y)$-Covariance of the random variables X and Y, $\rho_{X,Y}$ denotes the correlation coefficient between X and Y, *iid*-independent and identically distributed, *pdf*-probability density function, $B(n,p)$, $N(\mu,\sigma^2)$ and $U((a,b))$ denote respectively,the Binomial, the Normal and the Uniform distributions with the said parameters. $Rank(\mathbf{A})$ and $det(\mathbf{A})$ means rank and determinant respectively of the matrix $\mathbf{A}$. Members of $\mathbb{R}^n$ are column vectors $\underset{\sim}{\mathbf{x}}$ and $\underset{\sim}{\mathbf{0}}$ is the column vector of zeros or the zero vector.

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PART - A

1. Identify the odd one among the following:

   (A) All the irrational numbers in the interval $[4,5]$.

   (B) The set $\left\{ \sum_{j=1}^{\infty} \frac{d_j}{2^j}, \quad d_j \in \{0,1\}, \quad \forall j = 1,2,\ldots \right\}$

   (C) The subspace $\left\{ \begin{pmatrix} x \\ y \end{pmatrix}, \quad x+y=3 \right\}$ of $\mathbb{R}^2$.

   (D) The set of all the even powers of 2.

2. Two squares are selected at random from a chess board with equal probabilities, what is the probability that they do not have a common side?

   (A) 1/18 (B) 7/18 (C) 17/18 (D) 65/24.

3. $A = \{a_1, a_2, a_3, a_4\}, \quad B = \{b_1, b_2, b_3, b_4, b_5, b_6\}$How many one-one functions f exist from A to B?

   (A) 240. (B) 360. (C) 280. (D) 720.

4. X_1, X_2, X_3 and X_4 have the following distributions:

   i. $Pr(X_1 = j) = \frac{e^{-1}}{j!}, \quad j = 0,1,2,\ldots$

   ii. $Pr(X_2 = j) = \frac{3}{4}(\frac{1}{4})^j, \quad j = 0,1,2,\ldots$

   iii. $Pr(X_3 = n) = \frac{1}{27}\binom{n-1}{2}(\frac{2}{3})^{n-3}, \quad n = 3,\ldots$

   iv. $Pr(X_4 = j) = \frac{1}{(j+1)(j+2)}, \quad j = 0,1,2,\ldots$

   Which is the 'odd' random variable(not in terms of the range) among them?

   (A) X_1. (B) X_2. (C) X_3. (D) X_4.

5. Let $\mathcal{S}$ be the set of all 3×3 non singular real matrices. Identify the correct statement

   (A) $\mathcal{S}$ is a vector space over $\mathbb{R}$.

   (B) $\mathcal{S}$ is not a vector space over $\mathbb{R}$.

   (C) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \in \mathcal{S}$.

   (D) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix} \notin \mathcal{S}$.

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6. $\{A_n\}_{n=1}^{\infty}$ is the following sequence of subsets of $\mathbb{R}^2$: $A_n = \{(x,y), -n < x < n;\ 0 \le y \le \frac{1}{2n}\},\quad n = 1, 2, \ldots$.

(A) $\limsup_{n\to\infty} A_n = \emptyset$. (B) $\limsup_{n\to\infty} A_n = \{(0,0)\}$.

(C) $\liminf_{n\to\infty} A_n = \emptyset$. (D) $\liminf_{n\to\infty} A_n = \{(x,y) : -1 \le x \le 1 : 0 \le x \le 1\}$.

7. $X_1, \ldots, X_{40}$ are *iid* random variables that are Uniformly distributed over the interval $(0,1)$, $E(\frac{X_1+X_2+\ldots+X_{10}}{X_1+X_2+\ldots+X_{40}})$

(A) is equal to 1 (B)is equal to 1/2. (C) is equal to 1/4. (D)is equal to 0.

8. $X_1, X_2, \ldots, X_{10}$ are *iid* random variables that are exponentially distributed with mean λ. Consider the random variable $Y = \frac{X_2+X_3+X_5+X_8+X_9}{X_1+X_2+\ldots+X_{10}}$

(A) $Y \sim U((0,1/2))$. (B) $Y \sim \beta(5,10)$. (C) Y follows Cauchy distribution. (D) $Y \sim exp(1/2)$.

9. $\Omega = \{1,2,3,\ldots\}$, A = set of all even numbers, B = the set of all multiples of 3 and C = all prime numbers. Identify the correct statement.

(A) $C \subset A\triangle B$. (B) $30 \in A\triangle B$. (C) $45 \in A\triangle B$. (D) $81 \notin A\triangle B$.

10. What is the next term in the sequence

3 8 24 48 120 –?

(A) 168 (B)128 (C)152 (D)192

11. The probability of selecting a point inside a circle that is outside the largest square in the circle is

(A) less than 0.25. (B) between 0.25 and 0.5.

(C) between 0.5 and 0.7. (D) more than 0.75.

12. $Lim_{n\to\infty} n^2 \sum_{j=n+1}^{\infty} \frac{1}{j!}$

(A) does not exist. (B) is ∞. (C) is 1. (D) is 0.

13. $\mathbf{A} = \begin{pmatrix} 1/2 & 0 & 0 \\ 10 & 2 & 0 \\ 100 & 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 8 & 20 \\ 0 & 10 & 100 \\ 0 & 0 & 2 \end{pmatrix}$ the determinant of $\mathbf{A} \times \mathbf{B}$ is equal to

(A) 50. (B) 100. (C) 1. (D) 0.

14. Yesterday the price of an LPG cylinder was increased by 10% from the previous day's price, and today it was decreased by 5% from yesterday's price, compared to the price of the LPG cylinder day before yesterday, the price today is

(A) 10% less. (B) 5.5% more. (C) 5% more. (D) 4.5% more.

15. 20,000 candidates appeared for an exam, Ashok's percentile score is 87, it means

(A) Ashok scored 87% marks in the exam. (B) About 260 candidates performed worse th

(C) About 2600 candidates performed better than Ashok. (D) Ashok's rank in the exam is 87.

B - 2

16. Which of the following is a subspace in $\mathbb{R}^2$?

(A) $\left\{ \begin{pmatrix} x \\ y \end{pmatrix}, \quad y = 3x \right\}$. (B) $\left\{ \begin{pmatrix} x \\ y \end{pmatrix}, \quad x + y = 1 \right\}$.

(C) $\left\{ \begin{pmatrix} x \\ y \end{pmatrix}, \quad y = 2 + 3x \right\}$. (D) $\left\{ \begin{pmatrix} x \\ y \end{pmatrix} :, \quad x - y = 3 \right\}$.

17. Ashok and Bharat are two brothers, Ashok always tells lies and his messages are also lies, Bharat always speaks the truth and his messages are always true. They both use the same mobile phone, suppose you got a message from their phone saying, "Congratulations, you have got the job", if this is actually true, then upon asking them

(A) Both Ashok and Bharat will say that he did not send the message.

(B) Ashok will say that he sent it and Bharat will say that he did not.

(C) Both Ashok and Bharat will say that they sent the message.

(D) Ashok will say that he did not send it and Bharat will say that he sent it..

18. The determinant of 3×3 real symmetric matrix $\mathbf{A}$ is 8, further $det(\mathbf{A} - 2\mathbf{I}) = det(\mathbf{A} - \mathbf{I}) = 0$, the trace of $\mathbf{A}$

(A) is 6. (B) is 7. (C) is 8 (D) can not be determined based on the given information.

19. The value of $4x_1^2 - 4x_1x_2 + 3x_2^2$ is

(A) positive for every non - zero $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$

(B) 0 for every $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$.

(C) negative for every non - zero $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$.

(D) positive for some and negative for some non - zero $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$.

20. The first,second,...., and $n^{th.}$ columns of a $n \times n$ real matrix $\mathbf{A}$ are $\underset{\sim}{\mathbf{a}}_1, \ldots, \underset{\sim}{\mathbf{a}}_n$ respectively, if the rank of $\mathbf{A}$ is $r(< n)$, what is the rank of $\mathbf{B}$ whose first,second,...., and $n^{th.}$ columns are $\underset{\sim}{\mathbf{a}}_1, \underset{\sim}{\mathbf{a}}_1 + \underset{\sim}{\mathbf{a}}_2, \ldots, \underset{\sim}{\mathbf{a}}_1 + \ldots + \underset{\sim}{\mathbf{a}}_n$?

(A) n (B) $r - 1$. (C) r. (D) $n - 1$.

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21. $\underset{\sim}{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\underset{\sim}{y} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, the linear span of the set $S = \{\underset{\sim}{x}, \underset{\sim}{y}\}$ is

(A) the set $\left\{ \begin{pmatrix} a \\ a \end{pmatrix}, \forall a \in \mathbb{R} \right\}$.

(B) all the points on the the $X-$ axis only.

(C) all the points on the the line $y = -x/3$.

(D) all the points of $\mathbb{R}^2$.

22. Identify the correct statement regarding independence of events:

(A) A and B are independent events, A and C are independent events implies that B and C are independent.

(B) If A, B and C are pairwise independent, A and $B \cap C$ are also independent.

(C) If A and B are independent events, so are A^c and B^c.

(D) If A and B are independent events and $0 < P(B) < 1$, $P(A|B) < P(B)$

23. Of 100 M.Sc. students, 80 students can speak Telugu, 45 can speak Hindi, 50 can speak Bangla, 45 can speak both Hindi and Bangla, 40 can speak both Telugu and Hindi, while none can speak both Telugu and Bangla. How many of them can speak neither of the three languages?

(A) 10 (B) 90. (C) 20. (D) 80.

24. The function $f(x) = x - \lfloor x \rfloor, \quad x \geq 0$ is

(A) monotonically increasing in x.

(B) monotonically decreasing in x.

(C) continuous at every $x \geq 0$.

(D) bounded and not continuous at all positive integers.

25. Identify the correct order on the probability distribution of the $P(4)$, that is the Poisson random variable with parameter 4.

(A) $Pr(X = 1) < Pr(X = 2) < Pr(X = 4) < Pr(X = 6)$.

(B) $Pr(X = 7) < Pr(X = 6) < Pr(X = 5) < Pr(X = 4)$.

(C) $Pr(X = 2) < Pr(X = 4) < Pr(X = 6) < Pr(X = 9)$.

(D) $Pr(X = 7) > Pr(X = 4) > Pr(X = 8) > Pr(X = 3)$.

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26. To test if the assumption of homoscedasticity in the regression model of a variable with respect to a single regressor is valid we should plot

(A) the fitted values against the observed.

(B) the fitted values against the regressor values.

(C) the observed values against the regressor values.

(D) the residuals against the fitted values.

27. In 6 rolls or throws of a 6 faced die, the same number showing up in all the 6 rolls did not happen, this means

(A) different numbers showed up in each throw.

(B) 6 did not show up in any throw.

(C) 5 showed up in at least two throws.

(D) different numbers showed up in at least two throws.

28. If you have a large data of *iid* observations from a Uniformly distributed random variable, what percentage of data would you expect to be beyond one standard deviation from the median?

(A) A little over 40% but less than 50%.

(B) A little over 50% but less than 54%.

(C) A little over 57% but less than 63%.

(D) A little over 68% but less than 75%.

29. X is a random variable whose moment generating function exists, then,

(A) $E(|X|) \leq |E(X)|$. (B) $E(e^X) \geq e^{E(X)}$.

(C) $(E(X))^2 \geq E(X^2)$. (D) variance of X may not exist.

30. Which of the following statements is always correct?

(A) We do not need to randomize if our sample is sufficiently large.

(B) A large sample always ensures that it is representative of the population.

(C) In a properly chosen sample, an estimator will be less variable with a large sample and hence more reliable.

(D) In random samples, randomization ensures that we get precise and accurate estimates.

31. $X_i \sim exp(\lambda_i)$, $i = 1, 2, \ldots, n$ and are independent, then $E(Log \prod_{i=1}^{n}(1 - e^{-\lambda X_i}))$ is

(A) $-n$. (B) $2n$. (C) $-2n$. (D) 0.

32. Which of the following random variables has no mode?

(A)Poisson. (B)Binomial. (C)Exponentia.l (D)Uniform.

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33. Identify the correct statement: In regression analysis, a leverage point is

(A) one that is an outlier in the regressor space.

(B) one for which the response is an outlier.

(C) always influential.

(D) never influential.

34. A clinician wants to study how a treatment for diabetes is effected by age and weight of patients. What kind of experiment should she design?

(A) A CRD - completely randomized design with the some patients recieving the treatment and some getting no treatment.

(B) A 2^2 factorial design.

(C) A Block design.

(D) none of the above is suitable.

35. From a pack of cards, draw one card after another without replacing, the probability of getting 10 of spades in the 4^{th} draw is

(A) 1/52. (B) 1/4. (C) 1/13. (D) 1/10.

PART - B

36. Identify the correct ordering of the random variables with respect to their tail probabilities in increasing order: $X_i \sim N(50, 100)$, X_2 - Standard Cauchy random variable, $X_3 \sim t_5$- the central T random variable with parameter

(A) X_1, X_2, X_3. (B) X_1, X_3, X_2. (C) X_2, X_1, X_3. (D) X_2, X_3, X_1.

37. $X \sim exp(1)$, the random variable $Y = X - \lfloor X \rfloor$ follows

(A) the uniform distribution over (0,1), that is $Y \sim U((0,1))$.

(B) the $B(1 - e^{-1})$, that is $Pr(Y = 0) = e^{-1}$ and $Pr(Y = 1) = 1 - e^{-1}$.

(C) the beta distribution $\beta(1,1)$.

(D) the Geometric distribution with parameter $p = 1 - e^{-1}$.

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38. $\underset{\sim}{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_2\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}\right)$, that is $\underset{\sim}{X}$ is a bi-variate normal vector with the given parameters. Identify the correct statement or statements.

(I) The hypothesis: $\sigma_1^2 = \sigma_2^2$ is a simple hypothesis.

(II) The hypothesis: X_1 and X_2 are independent is not a simple hypothesis.

(III) The hypothesis: X_1 and X_2 are independent and identically distributed with standard deviations equal to 2 is a simple hypothesis.

(IV) The hypothesis: $\sigma_1^2 = 4, \sigma_2^2 = 5$ is a simple hypothesis.

(A) Only II and III are correct. (B) Only I and IV are correct.
(C) Only I is correct.. (D) All are correct

39. Identify the correct ordering of probabilities for $X \sim N(25, 49)$.

(A) $Pr(X \leq 23.25) < Pr(X > 27) < Pr(|X| > 3) < Pr(4 < X < 46)$.

(B) $Pr(X > 27) < Pr(|X| > 3) < Pr(4 < X46) < Pr(X \leq 23.5)$.

(C) $Pr(X \leq 23.25) < Pr(|X| > 3) < Pr(X > 27) < Pr(4 < X < 46)$.

(D) $Pr(X > 27) < Pr(X \leq 23.5) < Pr(4 < X < 46) < Pr(|X| > 3)$.

40. N is a non-negative integer valued random variable whose second moment exists, so

(A) $\sum_{i=0}^{\infty}(2i+1)Pr(X > i) = E(N^2)$. (B) $\sum_{i=0}^{\infty} iPr(X > i) = E(N^2)$.

(C) $\sum_{i=0}^{\infty} i^2 Pr(X > i) = E(N^2)$. (D) $\sum_{i=0}^{\infty} i^2 Pr(X \leq i) = E(N^2)$.

41. The random variable that follows $\beta(4, 2)$ distribution
(A) is skewed to the right. (B) is skewed to the left.
(C) is bi-modal. (D) is symmetric about its mean.

42. $X_1, X_2, \ldots$ are *iid* $U((-1, 1))$ random variables, which of the following statements is NOT true about the sequence $\{S_n\}_{n=1}^{\infty}$, $S_n = X_1 + \ldots + X_n$?

(A) $\frac{S_n}{n} \longrightarrow 0$ almost surely or with probability measure 1.

(B) The expected value of $Y = \sum_{j=1}^{\infty}(\frac{1}{3})^j X_j$ exists and is equal to 0.

(C) $\lim_{n\to\infty} Pr(\frac{S_n}{n} \leq x) \longrightarrow \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

(D) $\lim_{n\to\infty} Pr(\frac{S_n}{\sqrt{n}} \leq x) \longrightarrow H(x)$ where $H()$ is the distribution function of a normally distributed random variable with mean 0 and some variance.

43. For the model of $Y = 2 + 2X_1^2 - X_2^2 + X_1X_2$ for the variable Y based on the variables X_1 and X_2

(A) A change in X_1 from 4 to 5 or from 9 to 10 results in the same change in Y for every value of X_2.

(B) A change in X_1 from 4 to 5 results in different changes in Y, for different values of X_2.

(C) A change in X_1 from 4 to 5 or from 9 to 10 results in the same change in Y only if $X_2 = 0$.

(D) A change in X_2 from 2 to 3 results in the same change in Y for every value of X_1.

44. For the sequence of random variables $\{X_n\}_{n=1}^{\infty}$, let $\{\phi_n(t)\}$ be the corresponding sequence of characteristic functions, if $lim_{n\to\infty}\phi_n(t) = \phi(t), \quad \forall t \in \mathbb{R}$, where $\phi()$ is a characteristic function of some random variable X, then

(A) $X_n \longrightarrow X$ almost surely.

(B) $X_n \xrightarrow{P} X$, that is X_n converges in probability to X.

(C) $E(|X_n - X|) \longrightarrow 0$.

(D) $X_n \longrightarrow X$ in distribution.

45. Which of the following functions is NOT a characteristic function of any random variable

(A) $h_1(t) = \frac{2}{3} + \frac{e^{it}}{3}$. (B) $h_2(t) = e^{2it-\frac{t^2}{4}}$ (C) $h_3(t) = \frac{1}{2} + e^{it}$ (D) $h_4(t) = \frac{1}{1+t^2}e^{-|t|}$.

46. X and Y are independent Poisson random variables with means 3 and 6 respectively, $E(X|X+Y = 10)$ is

(A) 3/10. (B) 10. (C) 10/9. (D) 10/3.

47. Given that one heads showed up in three tosses of a coin for which probability of heads showing up in a toss is 2/5, what is the probability that heads occurred in the second toss?

(A) 1/5. (B) 1/3. (C) 3/25. (D) 2/5.

48. Which of the following random variables have real characteristic functions?

(I.) Exponential

(II.) Normal random variable with mean 0 and variance 64.

(III.) the uniform random variable in the interval $(-1, 1)$.

(IV.) The normal random variable with mean -10 and variance 64.

(A) only I and IV (B) only III and IV. (C) only I,II and IV. (D) only II and III.

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49. For a subset A of Ω define the indicator function of A as $I_A : \Omega \longrightarrow \{0,1\}$ as $I_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases}$.

Let $\Omega = (0,1]$ define a sequence of functions $f_n : \Omega \longrightarrow \{1,2,\ldots\}$ as $f_n(\omega) = nI_{(0,1/n]}(\omega)$, $n = 1,\ldots,$ identify the correct statement

(A) $\int_0^1 \lim_{n\to\infty} f_n(\omega)d\omega = 0.$

(B) $\lim_{n\to\infty} f_n(\omega) = 1, \ \forall\omega \in (0,1].$

(C) $\int_0^1 \lim_{n\to\infty} f_n(\omega)d\omega = \lim_{n\to} \int_0^1 f_n(\omega)d\omega.$

(D) $\int_0^1 f_n^2(\omega)d\omega = 1.$

50. Which of the following statements are always correct about a sequence of real valued random variables $\{X_n\}_{n=1}^{\infty}$ and another random variable X defined on a σ probability space $(\Omega, \mathcal{B}, P)$?

(I.) $X_n \longrightarrow X, a.s(P) \Rightarrow X_n \xrightarrow{P} X.$

(II.) $X_n \xrightarrow{P} X \Rightarrow X_n \longrightarrow X, a.s(P).$

(III.) $X_n \xrightarrow{P} X \Rightarrow X_n$ converges in distribution to X.

(IV.) If the probability distribution of X_n converges to the probability distribution of X, then $X_n \xrightarrow{P} X$.

(A) All of them (B) I and II only (C) I and III only. (D) I and IV only

51. X_1 and X_2 are independent random variables, identify the correct statements regarding the conditional distributions given $X_1 + X_2$.

I. If $X_i \sim P(\lambda_i)$, $i = 1,2$, the conditional distribution of X_1 given $X_1 + X_2 = n$ is $B(n, \frac{\lambda_1}{\lambda_1+\lambda_2})$.

II. If $X_i \sim N(\mu_i, \sigma_i^2)$, $i = 1,2$, $V(X_1|X_1 + X_2) < V(X_1)$.

III. If $X_i \sim B(n_i, p)$, $i = 1,2$, the conditional distribution of X_1 given $X_1 + X_2 = n(< n_1 + n_2)$ is $B(n, \frac{n_1}{n_1+n_2})$.

IV. If $X_i \sim B(n_i, p)$, $i = 1,2$, the conditional distribution of X_1 given $X_1 + X_2 = n(< n_1 + n_2)$ is a Hypergeometric distribution.

(A) All are correct (B)only II and III are correct

(C) only I, II and IV are correct. (D) only II, III and IV are correct

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52. A coin is tossed till the second heads shows up, if this happened in the 20^{th} toss,

(A) the probability that the first heads showed up in the 19^{th} toss is more than the probability that the first heads showed up in the second toss.

(B) the expected toss number in which the first heads showed up is less than 8.

(C) the first heads showed up in in any of the previous 19 tosses with equal probabilities if $1/19$ each.

(D) none of the above statements is correct.

53. At any metro station $0, 1, 2, 3, 4$, or 5 passengers board the train with probabilities $1/20$ each, $6, 7, 8, 9$ or 10 passengers will board with probabilities $7/50$ each. A passenger will alight at any intervening station with probability 0.2 independent of how many passengers are in the train or how many board at any station, the service begins at station 1, assuming that the train has infinite capacity, the expected number of passengers after station 2, but before stopping at station 3 is close to or equal to

(A) 9 (B) 7 (C) 15 (D) 11.

54. $\underset{\sim}{\mathbf{X}} \sim N_3\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\right),$

(A) $-(X_1^2 + X_2^2 + X_3^2) + X_1X_2 + X_1X_3 + X_2X_3$ and $(X_1 + X_2 + X_3)^2$ are independent.

(B) $-(X_1^2 + X_2^2 + X_3^2) + X_1X_2 + X_1X_3 + X_2X_3$ and $(X_1 - 2X_2 - X_3)^2$ are independent.

(C) $-(X_1^2 + X_2^2 + X_3^2) + X_1X_2 + X_1X_3 + X_2X_3$ and $X_1^2 + X_2^2 + X_3^2 - 2X_2X_3$ are independent.

(D) $-(X_1^2 + X_2^2 + X_3^2) + X_1X_2 + X_1X_3 + X_2X_3$ and $(X_1 - X_2)^2$ are independent.

55. A statistic is such that for 90% of the samples it takes values much more than a particular parameter, for 5% of the samples, it takes exactly the same value as the parameter and for the rest of the 5% of the samples it takes values that are slightly less than the parameter, this statistic

(A) is unbiased for the parameter as it overestimates some times and underestimates sometimes.

(B) has low mean squared error.

(C) not unbiased as it is more likely to overestimate the parameter.

(D) it is efficient.

56. Of N balls in a bag 5 are red, a draw of 10 balls from this bag had 4 red balls, the most likely number of balls from this among $\{21, 24, 25, 30\}$ is

(A) 21. (B) 24. (C) 25. (D) 30.

57. The probability of a coin showing up heads on tossing it is p. It showed up heads 3 times out of the 8 timed it was tossed. A MVU estimate for p^2 is

(A) 3/8 (B) 9/64 (C) 3/28. (D) 1/7.

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58. $X_1, \ldots, X_n$ is a random sample from the $U((-\theta, \theta))$, $\theta > 0$ distribution, let $X_{(1)}$ and $X_{(n)}$ denote the minimum and the maximum of the sample respectively, identify the correct statements regarding the statistic $T = max(-X_{(1)}, X_{(n)})$

(I.) T is a complete sufficient statistic for θ.

(II.) The $MVUE$-(Minimum variance unbiased estimator) for θ is $a_n T$ where a_n is a function of only the sample size.

(III.) T is a minimal sufficient, but not a complete sufficient statistic for θ.

(IV.) A MVUE does not exist for θ.

(A) only I and II (B) only I,II and III (C) only I and III (D) only IV

59. $X_1, \ldots, X_n$ is a random sample from the $N(\mu, \mu^2)$ population, identify the correct statements.

I. $\sum_{i=1}^{n} X_i$ is a sufficient statistic for μ.

II. $(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2)$ is a sufficient but not not complete sufficient statistic for μ.

III. $(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2)$ is minimal sufficient for μ.

IV. μ^2 has more than one unbiased estimator.

(A) all are correct.
(B) only I,II and III are correct
(C) only II,III and IV are correct.
(D) only I and IV are correct.

60. $X_1, \ldots, X_n$ is a random sample from the exponential random variable with mean $\frac{1}{\lambda}$, a unique $MVUE$ for λ

(A) does not exist.
(B) is $\frac{X_1+\ldots+X_n}{n}$.
(C) $\frac{n}{X_1+\ldots+X_n}$ is a $MVUE$.
(D) $\frac{n-1}{X_1+\ldots+X_n}$ is the unique $MVUE$.

61. To test $H_0 : f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, $\quad -\infty < x < \infty$ against $H_1 : f(x) = \frac{e^{-|x|}}{2}$, $\quad -\infty < x < \infty$, based on a single observation, the power of the most powerful test of size 2α, if $\Phi(z_c) = c$, $\ 0 \leq c \leq 1$ is

(A) $1 - e^{-z_\alpha}$. (B) e^{-z_α}. (C) $1 - \alpha$. (D) $1 - 2\alpha$.

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62. The 5% confidence region for the mean vector of the bivariate normal vector whose dispersion matrix is $\begin{pmatrix} 3 & 2 \\ 2 & 4 \end{pmatrix}$ is

(A) a circle whose center is the origin.

(B) a circle whose center is the sample mean vector.

(C) an ellipse whose centroid is the origin.

(D) an ellipse whose centroid is the sample mean vector.

63. For testing a simple Null Hypothesis against a simple Alternate hypothesis, Neyman - Pearson lemma gives

(A) a test of desired power and size for a given sample size.

(B) the most powerful test among all tests of a given size based on the given sample size.

(C) a test of desired size for a given sample size.

(D) the most powerful test of least size.

64. In a 2^3 factorial design with 4 replications, the number of observations are

(A) 16. (B) 32. (C) 8. (D) 12.

65. Which effects are confounded in the following layout of factor combinations in a replicate of 4 blocks in a 2^4 factorial design?

Block	*Combinations*
1	$(1), ab, cd, abcd$
2	ac, bc, ad, bd
3	a, b, acd, bcd
4	c, abc, d, abd

(A) $ABCD, CD$ and AB. (B) ABC, BC and A.

(C) BCD, ABC, and D. (D) ABD, ACD, and AC.

66. What are the block sizes - k and the number of blocks in which each pair of treatments appears - λ in a BIBD with 7 treatments, 7 blocks and each treatment appearing in 4 blocks?

(A) $k = 6,\ \lambda = 4$. (B) $k = 4,\ \lambda = 2$. (C) $k = 5,\ \lambda = 3$. (D) $k = 6,\ \lambda = 3$.

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67. The variable is effected by the random variables X_1, X_2 and X_3, the function $M(X_1, X_2, X_3)$ of X_1, X_2 and X_3 that has the least mean squared error with Y

(A) is always linear and and is positively correlated with Y.

(B) is the regression function $E(Y|X_1, X_2, X_3)$ and is positively correlated with Y.

(C) is always linear and and is perfectly correlated with Y.

(D) is the regression function $E(Y|X_1, X_2, X_3)$ and could be negatively correlated with Y.

68. In multiple linear regression if there is multicollinearity, the least squares estimators of the model parameters

(A) not efficient and small changes in the regressors can result in vast changes in the model parameter estimates.

(B) are unbiased and efficient.

(C) not unbiased.

(D) are robust.

69. Customers arrive at a bank in accordance with a Poisson process, if only one customer arrived in the first 10 minutes of the bank opening, the expected time at which this customer arrived is

(A) 9 minutes after opening of the bank.

(B) 8 minutes after opening of the bank.

(C) 6 minutes after opening of the bank.

(D) 5 minutes after opening of the bank.

70. The simple random walk $\{X_n = Y_1 + \ldots + Y_n \quad n = 1, 2, \ldots\}$ where $Pr(Y_i = 1) = 1/3.Pr(Y_i = -1) = 2/3, \quad i = 1, \ldots$ and independent is

(A) a positive recurrent Markov chain.

(B) a recurrent but not a positive recurrent Markov chain.

(C) a transient Markov chain.

(D) not an irreducible Markov chain.

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University of Hyderabad
Ph.D. Entrance Examinations - 2022

School/Department/Centre : Mathematics and Statistics
Course : Ph.D. Subject : Statistics

Q.No.	Answer	Q.No.	Answer	Q.No.	Answer
1	D	26	D	51	C
2	C	27	D	52	C
3	B	28	A	53	D
4	D	29	B	54	A
5	B	30	C	55	C
6	B	31	A	56	A
7	C	32	D	57	C
8	B	33	A	58	A
9	C	34	B	59	C
10	A	35	A	60	D
11	B	36	B	61	A
12	D	37	D	62	D
13	B	38	A	63	B
14	D	39	D	64	B
15	C	40	A	65	A
16	A	41	B	66	B
17	C	42	C	67	B
18	B	43	B	68	A
19	A	44	D	69	D
20	C	45	C	70	C
21	D	46	D		
22	C	47	B		
23	A	48	D		
24	D	49	A		
25	B	50	C		

Note/Remarks :

Signature
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