Z-6

ENTRANCE EXAMINATIONS-2021

M.Sc. Mathematics/Applied Mathematics

Time: 2 hours | PART A: 25 Marks

Max. Marks: 100 | PART B: 75 Marks

Hall Ticket Number										

Instructions

(1) Write your Hall Ticket Number on the OMR Answer sheet given to you. Also write the Hall Ticket Number in the space provided above.

(2) Answers to be marked on the OMR Answer Sheet.

(3) Please read the instructions carefully before marking your answers on the OMR Answer Sheet.

(4) Hand over the OMR Answer Sheet at the end of the examination to the Invigilator.

(5) No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.

(6) Calculators are not allowed.

(7) There are total of 50 questions in **PART A** and **PART B** together.

(8) There is a negative marking in PART A. Each correct answer carries 1 mark and each wrong answer carries -0.33 mark. Each question in PART A has only one correct option.

(9) There is no negative marking in PART B. Each correct answer carries 3 marks. Questions in PART B can have more than one correct option. If a question in this part has more than one correct option, then all the correct options have to be marked in OMR sheet to get 3 marks, otherwise zero marks will be credited for that question.

(10) The appropriate answer(s) should be coloured with either a blue or black ball point pen or a sketch pen. DO NOT USE A PENCIL.

(11) This booklet contains **13 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.

(12) The question paper can be taken away by the candidate at the end of the examination.

(13) **Notations:** $\mathbb{R}$ denotes the set of real numnbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Q}$ the set of rational numbers, $\mathbb{Z}$ the set of integers, $\mathbb{N}$ the set of natural numbers, and ϕ the empty set. For a set A, A^c denotes its complement. For a ring R and a positive integer n, $M_n(R)$ denotes the set of all $n \times n$ matrices with entries from R.

Part-A

(1) Suppose A is a nonzero 2×2 real matrix such that $\det(I + A) = 1 + \det(A)$, where I is the 2×2 identity matrix. Then which of the following statements is always true?

(A) The matrix A is singular

(B) $A^2 = A$

(C) $\lambda = 1$ is an eigenvalue of A

(D) If λ is an eigenvalue of A, then $-\lambda$ is also an eigenvalue of A

(2) Let A and B be 4×4 real matrices and $C = A^2 + AB$. If C is non-singular, then which of the following statements is always true?

(A) The trace of B is nonzero

(B) The trace of A is nonzero

(C) The matrix A is non-singular

(D) The matrix B is non-singular

(3) Together with $(1, 1, 0)$ and $(2, 2, 2)$, which of the following vector will form a basis of $\mathbb{R}^3$?

(A) $(3, 3, 3)$

(B) $(0, 0, 3)$

(C) $(-1, -1, 0)$

(D) $(1, 2, 0)$

(4) The area of $\{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 2\}$ is

(A) 4

(B) 8

(C) 12

(D) 16

(5) Let S' denote the set of all limit points of the nonempty set $S \subseteq \mathbb{R}$. Which of the following statements is true?

(A) If S is countably infinite then so is S'

(B) If S' is countably infinite then $S \cap S' = \phi$

(C) If S is countably infinite then S' is a finite set

(D) There exists a countably infinite set S such that S' is uncountable

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(6) Let $f, g : [0,1] \to \mathbb{R}$ be continuous. Define $u, v : [0,1] \to \mathbb{R}$ by $u(x) = \max\{f(x), g(x)\}$, $v(x) = \min\left\{\frac{f(x)}{1+|f(x)|}, \frac{g^2(x)}{\cosh(g(x))}\right\}$. Then

(A) both u and v are continuous

(B) both u and v are discontinuous

(C) u is continuous and v is discontinuous

(D) u is discontinuous and v is continuous

(7) Let $\alpha \in \mathbb{R}$ be an upper bound of a nonempty subset A of $\mathbb{R}$. Consider the following statements:

S_1 : The number α is the suprimum of A if β cannot be an upper bound of A for any $\beta < \alpha$.

S_2 : The number α is the suprimum of A if there exists $\epsilon > 0$ such that $\alpha - \epsilon$ cannot be an upper bound of A.

Then

(A) S_1 is true and S_2 is false

(B) S_1 is false and S_2 is true

(C) both S_1 and S_2 are true

(D) both S_1 and S_2 are false

(8) Let (a_n) be a sequence in $\mathbb{R}$. Then which of the following statements is true?

(A) If (a_n) is a monotonic sequence in $(0,1)$, then $(a_n) \to a$, for some $a \in (0,1)$

(B) If (a_n) is a Cauchy sequence and $|a_{2n} - \frac{1}{2}| < \frac{1}{2}$, $\forall n \in \mathbb{N}$, then $(a_n) \to a$, for some $a \in (0,1)$

(C) If (a_n) is a Cauchy sequence and $|a_p - \frac{1}{2}| < \frac{1}{4}$, $\forall$ prime number p, then, $(a_n) \to a$, for some $a \in (0,1)$

(D) If (a_n) is a Cauchy sequence then it is monotonic

(9) Let $f : (-1,1) \to \mathbb{R}$ be a non-constant continuous function. Consider the following statements.

S_1 : If f is differentiable at $x = 0$, then $\lim_{x\to 0} \frac{f(x)}{x}$ exists.

S_2 : If $\lim_{x\to 0} \frac{f(x)}{x}$ exists, then f is differentiable at $x = 0$, and the limit is equal to $f'(0)$.

Then

(A) S_1 is true and S_2 is false

(B) S_1 is false and S_2 is true

(C) both S_1 and S_2 are true

(D) both S_1 and S_2 are false

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(10) The ordinary differential equation representing the two parametric family of curves $y = \pm\sqrt{\alpha x + \beta}$, where α, β are the parameters, is

(A) $yy'' + y' = 0$

(B) $yy'' - (y')^2 = 0$

(C) $y'' + (y')^2 = 0$

(D) $yy'' + (y')^2 = 0$

(11) If $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{k}{x} N(x, y)$, for some $k \neq 1$, then which of the following functions is always an integrating factor of $M(x, y)dx + N(x, y)dy = 0$

(A) $\mu = x$

(B) $\mu = \frac{1}{x}$

(C) $\mu = x^k$

(D) $\mu = x^{-k}$

(12) Any solution of $x^2y'' + 3xy' + 10y = 0$, $x > 1$ is

(A) bounded

(B) has finite number of zeros

(C) monotonic

(D) always a constant function

(13) If $\bar{a}, \bar{b} : \mathbb{R}^3 \to \mathbb{R}^3$ are differentiable, then $\text{div}(\bar{a} \times \bar{b}) =$

(A) $\bar{a}.\text{curl}(\bar{b}) - \bar{b}.\text{curl}(\bar{a})$

(B) $\bar{b}.\text{curl}(\bar{a}) - \bar{a}.\text{curl}(\bar{b})$

(C) $\bar{a}.\text{curl}(\bar{b}) + \bar{b}.\text{curl}(\bar{a})$

(D) $\text{curl}(\bar{a}).\text{curl}(\bar{b})$

(14) The equation of the plane parallel to $4x + 2y + 7z + 25 = 0$ and passing through the point $(-1, -2, 7)$ is

(A) $4x + 2y + 7z + 4 = 0$

(B) $4x + 2y + 7z - 41 = 0$

(C) $4x + 2y - 7z + 57 = 0$

(D) $4x - 2y + 7z - 49 = 0$

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(15) Which of the following series converges?

(A) $\sum_{n=0}^{\infty} \frac{|\cos n| + |\sin n|}{n}$

(B) $\sum_{n=0}^{\infty} \left(1 - \frac{1}{n}\right)^n$

(C) $\sum_{n=0}^{\infty} \frac{n!}{n^n}$

(D) $\sum_{n=0}^{\infty} \frac{2^n}{\log n} \left(1 + \frac{1}{n}\right)^{-n}$

(16) The number of ways of arranging m boys and n girls, where $m \geq n$, around a circular table such that no two girls are next to each other is

(A) $\frac{(m-1)!m!}{(m-n)!}$

(B) $m!(n-1)!$

(C) $(m-1)!(n-1)!$

(D) $\frac{(n-1)!m!}{(m-n)!}$

(17) The equation of the straight line perpendicular to both $\frac{x-1}{3} = \frac{y+1}{-4} = \frac{z+1}{4}$ and $\frac{x+4}{3} = \frac{y-6}{-1} = \frac{z-1}{-5}$, and passing through $(1, 2, 3)$ is

(A) $\frac{x-1}{4} = \frac{y-2}{6} = \frac{z-3}{3}$

(B) $\frac{x-1}{8} = \frac{y-2}{19} = \frac{z-3}{1}$

(C) $\frac{x-1}{3} = \frac{y-2}{9}, z = 3$

(D) $\frac{x-1}{8} = \frac{y-2}{9} = \frac{z-3}{3}$

(18) If $(2, 5, 6)$ and $(4, 1, 2)$ are the end points of a diameter of a sphere, then the equation of the sphere is

(A) $x^2 + y^2 + z^2 - 6x - 6y - 8z + 25 = 0$

(B) $x^2 + y^2 + z^2 - 3x - 3y - 4z + 29 = 0$

(C) $x^2 + y^2 + z^2 - 6x - 6y - 8z + 29 = 0$

(D) $x^2 + y^2 + z^2 - 3x - 3y - 4z + 25 = 0$

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(19) If S denotes the sphere $x^2 + y^2 + z^2 = 3$, then $\int_S (2x^2 + 3y^2 - 4z^2) dS =$

(A) 6π

(B) 12π

(C) 54π

(D) 108π

(20) Which of the following statements is true?

(A) Every infinite cyclic group is isomorphic to $\mathbb{Z}$

(B) An infinite cyclic group may not have a non-trivial subgroup

(C) If p is a prime number, then any group of order p^2 is cyclic

(D) Every group of order 6 is isomorphic to $\mathbb{Z}/6\mathbb{Z}$

(21) Let G be a group of order 8. Then which of the following statements is true?

(A) G is necessarily abelian

(B) G is necessarily nonabelian

(C) There exists a nonabelian group of order 8

(D) There are exactly two nonisomorphic abelian groups of order 8

(22) Which of the following is a field?

(A) $\mathbb{Q}[X]/(X-1)(X-2)$

(B) $\mathbb{Z}[X]/(X^2+1)$

(C) $\mathbb{R}[X]/(X^2)$

(D) $\mathbb{Q}[X]/(X-100)$

(23) Let p is an odd prime number. The remainder of the number $1^p + 2^p + \cdots + p^p$ after division with p is

(A) 0

(B) 1

(C) -1

(D) $\frac{p-1}{2}$

(24) Let R be a ring with unity 1. Then the characteristic of R

(A) is always 0

(B) is always a prime number

(C) is always a composite number

(D) can be any non-negative integer

(25) Which of the following rings *does not* have a nonzero zero-divisor?

(A) 2×2 matrices over $\mathbb{R}$

(B) 2×2 matrices over $\mathbb{Z}/2\mathbb{Z}$

(C) polynomial ring $\mathbb{Z}[X]$

(D) $\mathbb{Z} \times \mathbb{Z}$

Part-B

(26) The distance of the point $(1, 2, 5)$ from the plane through $(1, -1, -1)$, having the normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-5}{3}$ and $\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z+7}{1}$ is

(A) $\sqrt{3}$

(B) $3\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{3}$

(27) A sphere has $x + y + z - 9 = 0$ and $x + y + z - 15 = 0$ as tangent planes. If the center of the sphere lies on the line $y = 2x, z = 3x$, then the equation of the sphere is

(A) $x^2 + y^2 + z^2 - 2x - 4y - 6z + 11 = 0$

(B) $x^2 + y^2 + z^2 - 2x - 4y - 6z + 5 = 0$

(C) $x^2 + y^2 + z^2 - 4x - 8y - 12z + 53 = 0$

(D) $x^2 + y^2 + z^2 - 4x - 8y - 12z + 47 = 0$

(28) The last two digits of $3^{3^{13}}$ are

(A) 27

(B) 81

(C) 43

(D) 61

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(29) Let V and W be two vector spaces. Let $T : V \to W$ be a linear transformation which is one-one, and $S : V \to W$ be an onto linear transformation. Assume that $A \neq \phi, B \neq \phi$ are subsets of V such that A is linearly dependent and B is linearly independent. Then which of the following statements are always true?

(A) Both $T(B)$ and $S(B)$ are linearly independent

(B) Both $T(A)$ and $S(A)$ are linearly dependent

(C) Both $S(A)$ and $S(B)$ are linearly dependent

(D) $T(A)$ is linearly dependent and $T(B)$ is linearly independent

(30) Let A be a 3×3 nonzero complex matrix. Let $\sigma(A)$ denote the set of eigenvalues of A. Then which of the following statements is true?

(A) If A is nilpotent, then $\sigma(A) = \{0, 1\}$

(B) If A is idempotent, then $\sigma(A) = \{0, 1\}$

(C) If $\sigma(A) = \{0\}$, then A is nilpotent

(D) If $\sigma(A) = \{0\}$, then A is idempotent

(31) Let V be a vector space. Suppose $\{u_1, u_2, \ldots, u_n\}$ and $\{w_1, w_2, \ldots, w_n\}$ are linearly independent and linearly dependent subsets of V respectively. Then there exists a linear transformation $T : V \to V$ such that

(A) $T(u_i) = w_i$

(B) $T(w_i) = u_i$

(C) $\mathrm{Ker}(T) = \mathrm{span}(\{u_1, u_2, \ldots, u_n\})$

(D) Range of $(T) = \mathrm{span}(\{u_1, u_2, \ldots, u_n\})$

(32) Let $f : [0, 1] \to \mathbb{R}$ be a function. Consider the following statements:
S_1: If f is differentiable in $(0, 1)$, continuous on $[0, 1]$, then f' is bounded in $(0, 1)$.
S_2: If f' exists and is bounded in $(0, 1)$, then f is uniformly continuous.
Then

(A) both S_1 and S_2 are true

(B) both S_1 and S_2 are false

(C) S_1 is true and S_2 is false

(D) S_1 is false and S_2 is true

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(33) Let $\mathcal{R}[a,b] = \{f : [a,b] \to \mathbb{R} / f$ is Riemann integrable$\}$. Then which of the following statements is/are true?

(A) If $f : [0,1] \to \mathbb{R}$ is such that $f\Big|_{[a,1]} \in \mathcal{R}[a,1]$, $\forall a \in (0,1)$, then $f \in \mathcal{R}[0,1]$

(B) If $f : [0,1] \to \mathbb{R}$ is continuous, then $f \in \mathcal{R}[0,1]$

(C) If $f \in \mathcal{R}[0,1]$, then there exists a finite set S such that f is continuous on $[0,1] \backslash S$

(D) If $f : [0,1] \to \mathbb{R}$ is monotone, then $\sin(f) \in \mathcal{R}[0,1]$

(34) Let (x_n), (y_n) be two sequences in $\mathbb{R}$. Define $z_n = \begin{cases} x_{\frac{n}{2}}, & \text{if } n \text{ is even,} \\ y_{\frac{n+1}{2}}, & \text{if } n \text{ is odd.} \end{cases}$

Then which of the following statements are true?

(A) If (x_n) and (y_n) are bounded, then (z_n) has a convergent subsequence

(B) If (x_n) and (y_n) are Cauchy sequences, then so is (z_n)

(C) If (z_n) is a Cauchy sequence, then so is (x_n)

(D). If (z_n) has a convergent subsequence, then at least one of (x_n) or (y_n) has a convergent subsequence

(35) Consider the following statements:
S_1 : If $f : \mathbb{Q} \to \mathbb{R}$ is continuous, then there exists a continuous function $F : \mathbb{R} \to \mathbb{R}$ such that $F(r) = f(r)$, $\forall r \in \mathbb{Q}$.
S_2 : If $F : \mathbb{R} \to \mathbb{R}$ is continuous and $f : \mathbb{Q} \to \mathbb{R}$ is defined by $F(r) = f(r)$, $\forall r \in \mathbb{Q}$, then f is continuous.

Then

(A) both S_1 and S_2 are true

(B) both S_1 and S_2 are false

(C) S_1 is true and S_2 is false

(D) S_1 is false and S_2 is true

(36) The value of α such that $(\frac{1}{x^2} + \frac{1}{y^2})dx + (\frac{\alpha x + 1}{y^3})dy = 0$ is exact, and the solution to the corresponding exact equation are

(A) $\alpha = 2$, $2(x^2 - y^2) - x = cxy^2$, where $c \in \mathbb{R}$ is an arbitrary constant

(B) $\alpha = -2$, $2(x^2 - y^2) - x = cxy^2$, where $c \in \mathbb{R}$ is an arbitrary constant

(C) $\alpha = 2$, $2x^2y^2 - x = cxy^2$, where $c \in \mathbb{R}$ is an arbitrary constant

(D) $\alpha = -2$, $2x^2y^2 - x = cxy^2$, where $c \in \mathbb{R}$ is an arbitrary constant

(37) If $x = e^{r\cos\theta}\cos(r\sin\theta)$, $y = e^{r\cos\theta}\sin(r\sin\theta)$, then

(A) $\dfrac{\partial x}{\partial\theta} = -r\dfrac{\partial y}{\partial r}$, $\dfrac{\partial y}{\partial\theta} = r\dfrac{\partial x}{\partial r}$

(B) $\dfrac{\partial x}{\partial\theta} = r\dfrac{\partial y}{\partial r}$, $\dfrac{\partial y}{\partial\theta} = r\dfrac{\partial x}{\partial r}$

(C) $\dfrac{\partial x}{\partial\theta} = -r\dfrac{\partial y}{\partial r}$, $\dfrac{\partial y}{\partial\theta} = -r\dfrac{\partial x}{\partial r}$

(D) $\dfrac{\partial x}{\partial\theta} = r\dfrac{\partial y}{\partial r}$, $\dfrac{\partial y}{\partial\theta} = -r\dfrac{\partial x}{\partial r}$

(38) Let $a_n, b_n > 0$, $a_n + b_n \neq 1$, $\forall n \in \mathbb{N}$, $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be convergent series. Consider the statements:

S_1 : The series $\sum_{n=1}^{\infty} a_n(1-b_n)^n$ converges.

S_2 : The series $\sum_{n=1}^{\infty} \dfrac{8^n(1+\frac{1}{n})^{4n}}{9^n(1-(a_n+b_n))^n}$ converges.

Then

(A) both S_1 and S_2 are true

(B) both S_1 and S_2 are false

(C) S_1 is true and S_2 is false

(D) S_1 is false and S_2 is true

(39) The solution of $\dfrac{dy}{dx} = \dfrac{2y}{x} + \dfrac{x^3}{y} + x\tan(\dfrac{y}{x^2})$, subject to $y(1) = 2\pi$ is

(A) $y\sin(\dfrac{y}{x^2}) - x\cos(\dfrac{y}{x^2}) = -x^3$

(B) $y\sin(\dfrac{y}{x^2}) + x\cos(\dfrac{y}{x^2}) = x^3$

(C) $y\sin(\dfrac{y}{x^2}) - x^2\cos(\dfrac{y}{x^2}) = -x^3$

(D) $y\sin(\dfrac{y}{x^2}) + x^2\cos(\dfrac{y}{x^2}) = x^3$

(40) We denote by $\simeq$ an isomorphism of groups. Which of the following is true?

(A) $(\mathbb{Z}, +) \simeq (\mathbb{Q}, +)$

(B) $(\mathbb{R}, +) \simeq (\mathbb{R}^{>0}, \cdot)$, where $\mathbb{R}^{>0}$ is the set of all nonzero positive reals

(C) $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \simeq S_3$, where S_3 is a permuation group on three letters

(D) $(\mathbb{Q}, +) \simeq (\mathbb{Q}^{>0}, \cdot)$, where $\mathbb{Q}^{>0}$ is the set of all positive rationals

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(41) The equation of the line that intersects the lines $x+y+z=1,\ 2x-y-z=2;\ \frac{x-1}{5}=\frac{y}{-1}=\frac{z+2}{6}$, and passes through the point $(1,1,1)$ is

$(A)\ \frac{x-1}{0}=\frac{y-1}{1}=\frac{z-1}{3}$

$(B)\ \frac{x-1}{1}=\frac{y-1}{0}=\frac{z-1}{3}$

$(C)\ \frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{1}$

$(D)\ \frac{x-1}{1}=\frac{y-1}{3}=\frac{z-1}{0}$

(42) Match the following vector spaces over $\mathbb{R}$ given in List-I with their dimensions given in List-II.

List-I	List-II
(i) The space of 4×4 real skew symmetric matrices	(a) 12
(ii) The space of 3×3 Hermitian matrices	(b) 6
(iii) The space of all continuous functions $f:\mathbb{R}\to\mathbb{R}$	(c) 9
such that $f(n)=0,\ \forall n\in\{1,2,3,4,5,6\}$	(d) ∞

$(A)\ (i)-(b),\ (ii)-(a),\ (iii)-(d)$

$(B)\ (i)-(b),\ (ii)-(c),\ (iii)-(d)$

$(C)\ (i)-(a),\ (ii)-(c),\ (iii)-(d)$

$(D)\ (i)-(a),\ (ii)-(b),\ (iii)-(c)$

(43) Match the following:

List-I	List-II
$(i)\ \lim_{n\to\infty}\left(2^{\frac{n+1}{n}}-1\right)^{n}$	(a) does not exist in $\mathbb{R}$
$(ii)\ \lim_{n\to\infty}\sum_{k=1}^{n}\frac{(-1)^k k^2}{1+k^2}$	(b) is equal to 1
$(iii)\ \lim_{x\to 0}\left(4+\frac{3}{x}\right)^{x}$	(c) is equal to 4

$(A)\ (i)-(b),\ (ii)-(a),\ (iii)-(c)$

$(B)\ (i)-(c),\ (ii)-(a),\ (iii)-(b)$

$(C)\ (i)-(a),\ (ii)-(b),\ (iii)-(c)$

$(D)\ (i)-(c),\ (ii)-(b),\ (iii)-(a)$

(44) Let $a_1, a_2 \in \mathbb{R}$. Match the properties of the roots of the quadratic equation $X^2 + a_1 X + a_2 = 0$ and that of the nonzero solutions of the differential equation $y'' + a_1 y' + a_2 y = 0$.

Roots	Nonzero solution
(i) Real, distinct, positive	(a) unbounded and monotonic in $[M, \infty]$ for some $M > 0$
(ii) Complex numbers with a positive real part	(b) bounded and periodic
(iii) purely imaginary	(c) unbounded and vanishes infinitely many times in $\mathbb{R}$

(A) $(i) - (a)$, $(ii) - (b)$, $(iii) - (c)$

(B) $(i) - (b)$, $(ii) - (c)$, $(iii) - (a)$

(C) $(i) - (a)$, $(ii) - (c)$, $(iii) - (b)$

(D) $(i) - (c)$, $(ii) - (a)$, $(iii) - (b)$

(45) We say that in a ring R with unity an element $a \in R$ is a *sum of squares* in R if $a = x_1^2 + x_2^2 + \cdots + x_n^2$ for some x_i's in R and for some natural number n. Then which of the following statements are true?

(A) -1 is a sum of squares in $\mathbb{R}$

(B) -1 is a sum of squares in $\mathbb{C}$

(C) -1 is a sum of squares in $\mathbb{Z}/17\mathbb{Z}$

(D) -1 is a sum of squares in $\mathbb{Q}[X]/(X-1)$

(46) Let R be a ring with unity 1. A nonzero element $e \in R$ is said to be an *idempotent* if $e \neq 1$ (and $e \neq 0$) and $e^2 = e$. Consider the following statements:
S_1 : The ring of matrices $M_2(\mathbb{R})$ has an idempotent.
S_2 : The ring $\mathbb{Z}/pq\mathbb{Z}$ (where p, q are distinct primes numbers) has an idempotent.
S_3 : The ring $\mathbb{Z}[X]/(X^2+1)$ has an idempotent.
Then

(A) only S_1 and S_3 are true

(B) only S_1 and S_2 are true

(C) only S_2 and S_3 are true

(D) only S_1 is true

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(47) Consider the following statements:

S_1 : Every subgroup of an abelian group is normal.

S_2 : If every subgroup of a group is normal, then the group is abelian.
Then

(A) both S_1 and S_2 are true

(B) both S_1 and S_2 are false

(C) S_1 is true and S_2 is false

(D) S_1 is false and S_2 is true

(48) If $x + y + \alpha z = 3$ is a tangent plane to $x^2 + y^2 + z^2 = 4$, then $\alpha =$

(A) $\pm\frac{1}{2}$

(B) $\pm\frac{3}{2}$

(C) $\pm\frac{2}{\sqrt{3}}$

(D) $\pm\frac{\sqrt{3}}{2}$

(49) Which of the following statements are true?

(A) $\mathbb{Q}$ is an ideal of $\mathbb{R}$

(B) A non-empty intersection of ideals in a ring is an ideal

(C) The set $\{(2m, n) : m, n \in \mathbb{Z}\}$ is an ideal of $\mathbb{Z} \times \mathbb{Z}$

(D) For a ring R, $\{0\} \subset R$ is an ideal of R

(50) Match the following groups with their properties:

Groups		Properties	
(i)	$\mathbb{Z}_2 \times \mathbb{Z}_2$	(a)	Cyclic
(ii)	$\mathbb{Z}$	(b)	Non-abelian and finite
(iii)	$M_2(\mathbb{R})$, the ring of 2×2 matrices	(c)	Abelian but not cyclic
(iv)	D_8 (Dihedral group)	(d)	Non-abelian and infinite

(A) $(i) - (a)$, $(ii) - (b)$, $(iii) - (c)$, $(iv) - (d)$

(B) $(i) - (b)$, $(ii) - (c)$, $(iii) - (d)$, $(iv) - (a)$

(C) $(i) - (c)$, $(ii) - (d)$, $(iii) - (a)$, $(iv) - (b)$

(D) $(i) - (c)$, $(ii) - (a)$, $(iii) - (d)$, $(iv) - (b)$

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University of Hyderabad
Entrance Examinations - 2021

School of Mathematics and Statistics
Course/Subject: M.Sc. in Mathematics/Applied Mathematics

Q.No.	Answer	Q.No.	Answer	Q.No.	Answer	Q.No.	Answer
1	D	26	A	51		76	
2	C	27	C	52		77	
3	D	28	A	53		78	
4	B	29	B,D	54		79	
5	D	30	C	55		80	
6	A	31	A, C, D	56		81	
7	A	32	B	57		82	
8	C	33	B,D	58		83	
9	B	34	A,C,D	59		84	
10	D	35	D	60		85	
11	C	36	B	61		86	
12	A	37	A	62		87	
13	B	38	A	63		88	
14	B	39	D	64		89	
15	C	40	B	65		90	
16	A	41	A	66		91	
17	D	42	B	67		92	
18	A	43	B	68		93	
19	B	44	C	69		94	
20	A	45	B,C	70		95	
21	C	46	B	71		96	
22	D	47	C	72		97	
23	A	48	A	73		98	
24	D	49	B, C, D	74		99	
25	C	50	D	75		100	

Note/Remarks :

Signature of the Head/Dean
School/Department/Centre