Y-61

ENTRANCE EXAMINATIONS – 2020
Ph.D. Mathematics/Applied Mathematics

Hall Ticket No.	

Time : 2 hours
Max. Marks : 70

PART A: 35 Marks
PART B: 35 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination to the Invigilator.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 70 questions in **PART A** and **PART B** together.
8. Each correct carries **1 mark**.
9. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. DO NOT USE A PENCIL.
10. This book contains **20 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.
11. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers, and $\mathbb{N}$ the set of all natural numbers.

Y-61

Part-A

1. Fill in the blank: 3, 8, 15, 24, ____

   A. 30
   B. 35
   C. 34
   D. 36

2. $10 \times 16 + 2 - 28 \div 2 = ?$

   A. 166
   B. -50
   C. 67
   D. 148

3. The sequence of symbols used in the place of circles to balance the equation $20 \bigcirc 10 \bigcirc 4 \bigcirc 12 \bigcirc 50 = 0$ is ____

   A. $\div, +, \times$ and $-$
   B. $\times, +, \div$ and $-$
   C. $-, \div, \times$ and $+$
   D. $-, +, \div$ and $\times$

4. Fill in the blanks in the following: Y, W, T, P, ____

   A. L
   B. K
   C. J
   D. I

5. Calculate the sum of the series: $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

   A. $\frac{1}{3}$
   B. $\frac{1}{2}$
   C. $\frac{3}{2}$
   D. $\frac{2}{3}$

6. What is the next term in the following? BKII, CMK, DON, _____

A. EPQ
B. EQP
C. EQQ
D. EQR

7. If 'CHAIR' is coded as 'AFYGP' then 'BMJJ' stands for _____

A. DOLL
B. MALL
C. TOSS
D. TALL

8. If 'PEN' is coded as 'QRFGOP' then how will you code 'CUT'?

A. DEWVUV
B. DEVVWU
C. DEUVVW
D. DEVWUV

9. If 'IVORY' is coded as '39251' and 'FAD' is coded as '467' then how will you code 'FAIRY'?

A. 46381
B. 46531
C. 46351
D. 43651

10. Choose the odd numeral pair:

A. 16:50
B. 9:29
C. 12:38
D. 6:21

11. How many three-letter words are there that start with any vowel and end with the letter 'S'?

A. 3380
B. 676
C. 32
D. 130

12. Suppose that two dice are rolled. What is the probability that there are different values on these two dice?

A. 2/3
B. 1/6
C. 5/6
D. 1/3

13. How many ways are there to form a committee of 3 women and 2 men from a set of 7 women and 4 men?

A. 200
B. 210
C. 168
D. 41

14. In n is a natural number, then $2\binom{n}{2} + n^2 =$ _____

A. $\binom{2n}{2}$
B. $\binom{2n+1}{2}$
C. $\binom{2n}{2} + \binom{n}{2}$
D. $\binom{n^2}{2}$

15. If 'GFAH' is coded as 'LKFM' then 'RSKB' is coded as _____

A. WXPG
B. WPXG
C. WXPH
D. WYPG

Y-61

16. If A and B are any sets then $B \setminus (B \setminus A) =$ _____

A. ϕ

B. A

C. $A \cap B$

D. $A \cup B$

17. Let A and B be any two sets and let $A \Delta B$ be the symmetric difference of A and B. Then $A \Delta B = \phi$ if and only if _____

A. $A = \phi$

B. $B = \phi$

C. $A \subset B$

D. $A = B$

18. Let A and B be any two sets and let $\tilde{B}$ be the complement of B. Then $(A \cap B) \cup (A \cap \tilde{B}) =$ _____

A. A

B. B

C. ϕ

D. $A \cup B$

19. At 3:40, the hour hand and the minute hand of a clock form an angle of _____

A. 130 degrees

B. 120 degrees

C. 150 degrees

D. 140 degrees

20. If in a certain code, 'GLAMOUR' is written as 'IJCNMWP' and 'MISRULE' is written 'OGUSSNC', then how will 'TOPICAL' be written in that code?

A. VMRJEUN

B. VMRHACJ

C. VMRJACJ

D. VNRJABJ

Y-61

21. Find the missing number in the series $10, 67, 341, 1369,$ ____.

A. 7311

B. 4111

C. 5665

D. 8576

22. The negation of the statement "If I eat icecream, then I am happy" is

A. I eat icecream and I am not happy.

B. I eat icecream and I am happy.

C. I eat icecream or I am happy.

D. I do not eat icecream or I am happy.

23. The number of bijections of the set $\{1, 2, 3, \ldots, n\}$ is

A. $n - 1$

B. n

C. $n!$

D. $(n + 1)!$

24. Which of the following is a finite set?

A. The set of natural numbers.

B. The set of all prime numbers.

C. The set of all words in every dictionary on Earth.

D. All functions from $\mathbb{N} \to \{0, 1\}$.

25. Let cardinality of A, $|A| = n$ and that of a set B, $|B| = m$. If $n < m$, then the number of injective functions from A to B is

A. m^n

B. 2^m

C. $n!$

D. $m(m-1)\cdots(m-n+1)$

Y-61

26. Let $f: A \to B$ and $g: B \to A$ be such that $g \circ f = I_A$, where I_A is the identity function on A. Then,

A. f is surjective and g is injective.
B. f is injective and g is surjective.
C. f is injective but g is may not be surjective.
D. f may not be injective and g may not be surjective.

27. Consider the expression $1 - \frac{4}{5} - \frac{4}{5^2} - \frac{4}{5^3} - \cdots - \frac{4}{5^n} < \frac{1}{600}$. Then

A. There is no natural number n satisfying the above expression.
B. $n = 4$
C. $n = 3$
D. $n = 2$

28. Suppose that P, Q, and R are statements. Assume that P implies Q, and that Q implies R. If we also know that P is false, then

A. Q is false.
B. R is false.
C. R implies P.
D. No conclusion can be drawn.

29. Let A, B be nonempty finite sets. For a set S we denote by $|S|$ the cardinality of S. We denote by $A \setminus B = \{x \in A \text{ and } x \notin B\}$ and $B \setminus A$ is similarly defined. Consider the following statements, and choose the correct option(s).

1. If $|A \setminus B| = |B \setminus A|$ then, $|A| = |B|$
2. $|A \cup B| = |A| + |B|$
3. $|A \times \{1\}| > |A|$
4. $|A \cup B| > |A|$ for any nonempty sets A, B.

A. Only 1 is correct.
B. Only 1 and 3 are correct.
C. Only 2 is correct.
D. Only 4 is correct.

30. Suppose that P and Q are statements. To disprove the claim that 'Both P and Q are true', which of the statement(s) below we need to show? Choose the correct alternative(s).

1. At least one of P and Q is false.
2. Both P and Q are false.
3. P does not imply Q, and Q does not imply P.
4. P is false if and only if Q is true.

A. Only 2 is correct
B. Only 3 is correct
C. Only 3 and 4 are correct
D. Only 1 is correct

31. Let $f: X \to Y$ be a function and $\{A_i\}_{i \in \mathbb{N}}$ be a nonempty collection of nonempty subsets of X. Let B, C be nonempty subsets of Y. Consider the following statements, and choose the correct alternative(s).

1. $f(\bigcap_i A_i) = \bigcap_i f(A_i)$
2. $f(\bigcup_i A_i) = \bigcup_i f(A_i)$
3. $f(\bigcap_i A_i) \subseteq \bigcap_i f(A_i)$
4. $f^{-1}(B \cap C) = f^{-1}(B) \cap f^{-1}(C)$.

A. Only 1, 2 and 3 are correct.

B. Only 1 and 2 are correct.

C. Only 2, 3 and 4 are correct.

D. Only 1, 3 and 4 are correct.

32. In this question, the left hand column consists of real valued functions on $\mathbb{R}$.

1. $f(x) = e^x$	I. injective
2. $f(x) = \sin x$	II. bijective
3. $f(x) = x - 1$	III. neither injective nor surjective

We say that the pair is 'matched' if the function satisfies a property listed under the second column. Which of the pairs are matched?

A. (I,1), (II,3) and (III, 2)

B. (I,1), (II,2) and (III,3)

C. (I,2), (II,3) and (III,1)

D. (I,3), (II,2) and (III,1)

33. In the following, the left hand column consists of sets A_i and a relation R_i on A_i. While the right hand column consists of a type of a relation.

1. $A_1 = \{1, 2\}$, $R_1 = \{(1, 1), (2, 2)\}$	I. Only symmetric
2. $A_2 = \mathbb{N}$, $R_2 = \{(x, y) : x + y \text{ is a multiple of } 3\}$	II. Equivalence relation
3. $A_3 = \{1, 2, 3\}$, $R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$	III. Only transitive
4. $A_4 = \{1, 2, 3\}$, $R_4 = \{(1, 2), (1, 3), (2, 3)\}$	IV. Only reflexive

We say that the pair is 'matched' if the relation R_i listed on the left hand column satisfies a type of a relation listed in the right hand column. Which of the pairs are matched?

A. (I,2), (II,4), (III,1), (IV,3)

B. (I,2), (II,1), (III,3), (IV,4)

C. (I,1), (II,2), (III,4), (IV,3)

D. (I,2), (II,1), (III,4), (IV,3)

Y-61

34. Let $\mathcal{P}(S)$ be the power set of a set S, and $|S|$ denotes the cardinality of S. Then,

A. $|(\mathcal{P}(\mathbb{R}))| > |(0,1)| > |\mathbb{R} \setminus \mathbb{Q}| = |\mathbb{Q}| = |\mathbb{Z}|$
B. $|(\mathcal{P}(\mathbb{R}))| = |\mathbb{R} \setminus \mathbb{Q}| > |(0,1)| > |\mathbb{Q}| = |\mathbb{Z}|$
C. $|(\mathcal{P}(\mathbb{R}))| = |(0,1)| = |\mathbb{R} \setminus \mathbb{Q}| > |\mathbb{Q}| > |\mathbb{Z}|$
D. $|(\mathcal{P}(\mathbb{R}))| > |(0,1)| = |\mathbb{R} \setminus \mathbb{Q}| > |\mathbb{Q}| = |\mathbb{Z}|$

35. Consider the positive real numbers $\sqrt{2}$, $\sqrt[3]{3}$, and $\sqrt[6]{6}$. Then which order among the following is correct?

A. $\sqrt[6]{6} < \sqrt{2} < \sqrt[3]{3}$
B. $\sqrt{2} < \sqrt[6]{6} < \sqrt[3]{3}$
C. $\sqrt[6]{6} < \sqrt[3]{3} < \sqrt[2]{2}$
D. $\sqrt{2} < \sqrt[3]{3} < \sqrt[6]{6}$

TURN THE PAGE FOR PART B →

Y-61

Part-B

36. Let f be a real valued function defined on an open subset of $\mathbb{R}$. Which of the following statements is/are true?

I. If both f^2 and $|f|$ are measurable, then f is measurable.
II. If $f > 0$, and f^2 is measurable, then f is measurable.
III. If $|f|$ is measurable, then f is measurable.
IV. If for all $c \in \mathbb{R}$, $\{x : f(x) \leq c\}$ is a measurable set, then f is measurable.

A. IV
B. II, III and IV
C. I, II and IV
D. II and IV

37. Let A, B be two measurable subsets of $\mathbb{R}$, and let m denote the Lebesgue measure on $\mathbb{R}$. Which of the following statements is/are true?

I. If $m(A \cup B) = m(A)$, then $m(B) = 0$.
II. If $m(A) = 0$, then $m(A \cup B) = m(B)$.
III. "$m(A) = 0 \implies m(A \cup B) = m(B)$" is true, if $m(B)$ is finite.
IV. $m(A \cup B) = m(A) + m(B)$ is true, only if A and B are disjoint sets.

A. II and III
B. I
C. IV
D. I, II and III

38. Which of the following are TRUE?

(1) Every countable subset of $\mathbb{R}$ is Lebesgue measurable.
(2) Every dense open subset of $(0, 1)$ has Lebesgue measure equal to 1.
(3) The Lebesgue measure of middle-third Cantor set is positive.
(4) Every continuous function $f : \mathbb{R} \to \mathbb{R}$ is Borel measurable.

A. (2) and (4)
B. (1) and (3)
C. (2) and (3)
D. (1) and (4)

Y-61

39. Consider the following statements:

I. An entire function bounded outside a compact set is constant.
II. An entire function bounded outside a bounded set is constant.

Determine the correct option(s).

A. Only I is true
B. Only II is true
C. Both I and II are true
D. Both I and II are false

40. Let $f : [a, b] \to \mathbb{R}$ be continuous and differentiable on (a, b). Consider the following statements.

I. Intermediate value theorem holds for f on (a, b).
II. Intermediate value theorem holds for f' on (a, b).

Choose the correct answer.

A. Both I and II are true
B. I is true but not II
C. II is true but not I
D. I and II are both false

41. Let R be a nonzero ring with unity 1. Consider the following statements.

I. If $r, s \in R \setminus \{0\}$ are two zero-divisors, then $r + s$ is a zero-divisor.
II. If $r, s \in R \setminus \{0\}$ are two nilpotent elements such that $rs = sr$, then $r + s$ is nilpotent.
III. If $r, s \in R \setminus \{0\}$ are two nilpotent elements, then $r + s$ is nilpotent.

Which of them is/are correct?

A. Only I is correct
B. Only II is correct
C. Only I and II are correct
D. Only II and III are correct

Y-61

42. Which of the following is/are true?
    (1) Continuous image of a locally compact space is locally compact.
    (2) Continuous image of a locally connected space is locally connected.

A. Only (1)

B. Only (2)

C. Both (1) and (2)

D. Neither (1) nor (2)

43. Which of the following is true?
    (1) Every compact space is locally compact.
    (2) Every connected space is locally connected.

A. Only (1)

B. Only (2)

C. Both (1) and (2)

D. Neither (1) nor (2)

44. A *topological property* or *topological invariant* is a property of a topological space which is invariant under homeomorphisms.
    I. Separable II. Completeness III. Regular IV. Metrizable
    Pick up the property which are topological invariants from the above:

A. I and II

B. I, III and IV

C. II and IV

D. II, III and IV

45. Let $f : (0,1) \to \mathbb{R}$ be any function. Which of the following statements is/are correct?
    I. If f is bounded, then it is continuous.
    II. If f is continuous, then it is bounded.
    III. If f is strictly increasing, then it is continuous.
    IV. If f is differentiable, then f is continuous.

A. I and II

B. III and IV

C. IV

D. III

46. Let X be a normed linear space and $T : X \to X$ be a linear transformation. Which of the following statements is/are correct?

I. T is bounded iff T is continuous.

II. If T is compact, then T is continuous.

III. If T is bounded, then T is compact.

IV. If X is finite dimensional and T is continuous, then T is compact.

A. I and II

B. I, II and IV

C. II and IV

D. II

47. Which of the following pairs are correctly matched?

I. $f(x) = \|x\|$	1. differentiable at no point
II. $f(x) = e^x$	2. differentiable everywhere
III. $f(x) = [x]$	3. not differentiable at some points
IV. $f(x) = \begin{cases} 0 \ if \ x \in \mathbb{Q} \\ 1 \ if \ x \notin \mathbb{Q} \end{cases}$	4. not differentiable at infinitely many points

A. (I,1), (II,2), (III,3), (IV,4)

B. (I,4), (II,3), (III,1), (IV,2)

C. (I,3), (II,2), (III,4), (IV,1)

D. (I,3), (II,2), (III,1), (IV,4)

48. Which of the following groups are correctly matched with their types?

I. $\mathbb{Z}_4$	1. non abelian
II. $\mathbb{Z}_4 \oplus \mathbb{Z}_2$	2. cyclic
III. S_5	3. simple
IV. A_{10}	4. abelian

A. (I,4), (II,2), (III,3), (IV,1)

B. (I,2), (II,4), (III,1), (IV,3)

C. (I,4), (II,2), (III,1), (IV,3)

D. (I,4), (II,1), (III,2), (IV,3)

4-61

49. Which of the following sequences are correctly matched?

I. $1, 2, 3, 4, \ldots$	1. bounded and not monotonic
II. $1, 2, 1, 2, \ldots$	2. monotonic and bounded
III. $1, \frac{1}{2}, \frac{1}{3}, \ldots$	3. monotonic and unbounded
IV. $1, 2, 1, 3, 1, 4, \ldots$	4. unbounded and not monotonic

A. (I,3), (II,1), (III,2), (IV,4)

B. (I,3), (II,4), (III,2), (IV,1)

C. (I,3), (II,2), (III,1), (IV,4)

D. (I,1), (II,2), (III,3), (IV,4)

50. Match the following:

(a). Aut $\mathbb{Z}$	(p). $\{1, -1\}$
(b). $U(9)$	(q). 1
(c). Units of $\mathbb{Z}[\sqrt{-2}]$	(r). $\mathbb{Z}_2$
(d). Characteristic of zero ring	(s). $\{1, 5, 7, 9\}$

A. (a,r), (b,q), (c,s), (d,p)

B. (a,q), (b,p), (c,s), (d,r)

C. (a,r), (b,s), (c,q), (d,p)

D. (a,r), (b,s), (c,p), (d,q)

51. Match the following:

(a) $SL_n(\mathbb{R})$, the set of all $n \times n$ matrices with determinant 1	(1) not connected, not compact
(b) $GL_n(\mathbb{R})$, the set of all $n \times n$ invertible matrices	(2) path connected, not compact
(c) $O_n(\mathbb{R})$, the set of all $n \times n$ orthogonal matrices	(3) path connected and compact
	(4) compact, not connected

A. $a \leftrightarrow 1$, $b \leftrightarrow 2$, $c \leftrightarrow 4$

B. $a \leftrightarrow 3$, $b \leftrightarrow 1$, $c \leftrightarrow 2$

C. $a \leftrightarrow 2$, $b \leftrightarrow 1$, $c \leftrightarrow 4$

D. $a \leftrightarrow 4$, $b \leftrightarrow 2$, $c \leftrightarrow 3$

52. Match the following:

(a) The number of group automorphisms on $\mathbb{Z}_8$ is	(1) 1
(b) The number of abelian groups of order 12 is	(2) 2
(c) The number of Sylow 2-subgroups of a group of order 24 is either 1 or	(3) 3
(d) The number of simple groups of order 60 is	(4) 4

A. $a \leftrightarrow 4,\ b \leftrightarrow 1,\ c \leftrightarrow 3,\ d \leftrightarrow 2$

B. $a \leftrightarrow 4,\ b \leftrightarrow 2,\ c \leftrightarrow 3,\ d \leftrightarrow 1$

C. $a \leftrightarrow 1,\ b \leftrightarrow 4,\ c \leftrightarrow 2,\ d \leftrightarrow 3$

D. $a \leftrightarrow 4,\ b \leftrightarrow 1,\ c \leftrightarrow 2,\ d \leftrightarrow 3$

53. Match the following polynomial elements to the rings in which they are units.

I. $3+14X$	1. $\mathbb{Z}_{18}[X]$
II. $1+3X+27X^2$	2. $\mathbb{Z}_{28}[X]$
III. $5+6X+12X^3$	3. $\mathbb{Z}_{81}[X]$
	4. $\mathbb{Z}_{98}[X]$

A. (I,2), (II,4), (III,1)

B. (I,3), (II,2), (III,1)

C. (I,2), (II,3), (III,1)

D. (I,1), (II,2), (III,3)

54. Which of the following pair of statements are correctly matched? (Each of the following f is a function $\mathbb{Z}_{50} \to \mathbb{Z}_{20}$.)

I. $f(x)=4x$	1. is not a group homomorphism
II. $f(x)=5x$	2. is a group homomorphism but not ring homomorphism
III. $f(x)=16x$	3. is a surjective group homomorphism
	4. is a ring homomorphism

A. (I,1), (II,2), (III,3)

B. (I,2), (II,3), (III,4)

C. (I,2), (II,1), (III,4)

D. (I,4), (II,2), (III,1)

55. Let $\phi(x)$ be an extremal to the functional $J[y] = \int_a^b F(x, y(x), y'(x))dx$. Let α and β be two arbitrary constants. Then match the following functionals in the left column with the corresponding extremals in the right column:

(1) $F = x^2 + y^2$	(i) $\phi \equiv 0$
(2) $F = y' + \sin(y')$	(ii) $\phi = \alpha \log x + \beta$
(3) $F = x(y')^2$	(iii) $\phi = \alpha x + \beta$

A. (1)-(iii), (2)-(i), (3)-(ii)
B. (1)-(i), (2)-(iii), (3)-(ii)
C. (1)-(ii), (2)-(iii), (3)-(i)
D. (1)-(iii), (2)-(ii), (3)-(i)

56. Match the following PDEs in the left column with the corresponding types in the right column:

(1) The wave equation	(i) elliptic
(2) The heat equation	(ii) hyperbolic
(3) The Laplace equation	(iii) parabolic

A. (1)-(iii), (2)-(i), (3)-(ii)
B. (1)-(ii), (2)-(i), (3)-(iii)
C. (1)-(ii), (2)-(iii), (3)-(i)
D. (1)-(iii), (2)-(ii), (3)-(i)

57. Match the following system of differential equations with the corresponding type of critical point at the origin

(a) $\frac{dx}{dt} = x, \ \frac{dy}{dt} = -x + 2y$	(i) node.
(b) $\frac{dx}{dt} = -3x + 4y, \ \frac{dy}{dt} = -2x + 3y$	(ii) saddle point.
(c) $\frac{dx}{dt} = -y, \ \frac{dy}{dt} = x$	(iii) centre.
(d) $\frac{dx}{dt} = -x - 2y, \ \frac{dy}{dt} = 4x - 5y$	(iv) spiral.

A. (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
B. (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)
C. (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
D. (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

4-61

58. Consider the integral equation $\phi(x) = f(x) + \lambda \int_{-1}^{1} xe^t \phi(t)dt$. Then match the following

(i) $\lambda = e,\ f(x) = x$	(a) Infinite number of solutions.
(ii) $\lambda = \mathrm{e}/2,\ f(x) = x\mathrm{e}^{-x}$	(b) No solution.
(iii) $\lambda = \mathrm{e}/2,\ f(x) = \mathrm{e}^{-x}$	(c) unique solution.

A. (i)-(c), (ii)-(a), (iii)- (b)

B. (i)-(a), (ii.)-(b), (iii.)- (c)

C. (i)-(b), (ii.)-(c), (iii.)- (a)

D. (i)-(c), (ii.)-(b), (iii.)- (a)

59. Arrange the numbers in decreasing order: e, π, e^{π}, π^{e}.

A. π^e, e^π, π, e

B. e^π, π^e, e, π

C. e^π, π^e, π, e

D. π^e, e^π, e, π

60. Arrange the following sets in the increasing order of inclusion. A- set of all abelian groups, I- set of all inner product spaces, N- set of all normed linear spaces, B- set of all vectors spaces.

A. A, N, I, B

B. I, N, A, B

C. N, I, B, A

D. I, N, B, A

61. Arrange the following rings in the inclusion order

I. Integral Domain

II. Unique factorization domain

III. Commutative ring

IV. Euclidean domain

A. IV, I, II, III

B. IV, II, I, III

C. II, IV, I, III

D. I, II, IV, III

4 - 61

62. For a function $f : [0,1] \to \mathbb{R}$, arrange the following properties in the increasing order of strength: (1) f is Lipschitz continuous. (2) f is continuous at infinitely many points. (3) f is uniformly continuous. (4) f is Riemann integrable. (5) f is continuously differentiable. (6) f is absolutely continuous.

A. 4,2,3,1,6,5

B. 2,4,3,6,1,5

C. 4,2,3,6,5,1

D. 2,4,3,1,5,6

63. Arrange the following in the increasing order of the number of connected components:

(1) $\mathbb{R}^2 \setminus (\mathbb{Q} \times \mathbb{Q})$
(2)$\{(x, y) \in \mathbb{R}^2 : xy = 1\}$
(3) $\mathbb{Q}$

A. 1, 2, 3

B. 1, 3, 2

C. 2, 3, 1

D. 3, 2, 1

64. Arrange the following field extensions in the increasing order of their degrees:

(1) $\mathbb{Q}(\sqrt{5} + \sqrt{3})$ over $\mathbb{Q}$
(2) $\mathbb{Q}(\pi)$ over $\mathbb{Q}$
(3) $\mathbb{Q}(i)$ over $\mathbb{Q}$

A. 1, 3, 2

B. 3, 1, 2

C. 3, 2, 1

D. 3, 1, 2

65. Consider the cyclic group of units $G_1 = \mathcal{U}(\mathbb{Z}_{23}), G_2 = \mathcal{U}(\mathbb{Z}_{27}), G_3 = \mathcal{U}(\mathbb{Z}_9), G_4 = \mathcal{U}(\mathbb{Z}_{25})$ (these are group of units of integers modulo corresponding n). The arrangement of these groups in decreasing order of the number of generators is:

A. G_1, G_2, G_4, G_3
B. G_1, G_4, G_2, G_3
C. G_3, G_2, G_1, G_4
D. G_1, G_4, G_3, G_2

66. The arrangement of the groups $G_1 = \mathbb{Z}_{36}, G_2 = \mathbb{Z}_{30}, G_3 = \mathbb{Z}_{48}, G_4 = \mathbb{Z}_{60}$ according to increasing number of nilpotent elements in them, is:

A. G_2, G_1, G_3, G_4
B. G_4, G_3, G_1, G_2
C. G_2, G_4, G_1, G_3
D. G_4, G_1, G_2, G_3

67. Find the correct order of the following phrases so that it means that a given function $f : \mathbb{R} \to \mathbb{R}$ is not uniformly continuous.

(a) for every $\delta > 0$

(b) there exist $x, y \in \mathbb{R}$ with $|x - y| < \delta$

(c) there exists $\epsilon > 0$ such that

(d) such that $|f(x) - f(y)| > \epsilon$

A. 1, 3, 2, 4
B. 1, 4, 3, 2
C. 3, 1, 2, 4
D. 3, 2, 1, 4

68. Let $V(n)$ denote the volume of the ball in $\mathbb{R}^n$ with center at the origin and radius 1. Then the arrangement of the numbers $V(2), V(10), V(100), V(1000)$ in the ascending order is:

A. $V(2)$, $V(10)$, $V(100)$, $V(1000)$
B. $V(1000)$, $V(100)$, $V(10)$, $V(2)$
C. $V(100)$, $V(10)$, $V(1000)$, $V(2)$
D. $V(1000)$, $V(10)$, $V(2)$, $V(100)$

69. Let u be the solution of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$ satisfying $u(x,y) = 5$ whenever $x^2 + y^2 = 4$. Then the arrangement of the numbers $u(3,4)$, $u(5,5)$, $u(0,1)$, $u(1,3)$ in the descending order is:

A. $u(5,5)$, $u(3,4)$, $u(0,1)$, $u(1,3)$

B. $u(0,1)$, $u(5,5)$, $u(1,3)$, $u(3,4)$

C. $u(0,1)$, $u(3,4)$, $u(5,5)$ $u(1,3)$

D. $u(5,5)$, $u(3,4)$, $u(1,3)$, $u(0,1)$

70. Let $\phi(x)$ be the solution of the integral equation
$x = \int_0^x e^{(x-t)}\phi(t)dt$. Then

A. $\phi(0) \leq \phi(1) \leq \phi(2) \leq \phi(3)$

B. $\phi(1) \leq \phi(0) \leq \phi(3) \leq \phi(2)$

C. $\phi(2) \leq \phi(3) \leq \phi(0) \leq \phi(1)$

D. $\phi(3) \leq \phi(2) \leq \phi(1) \leq \phi(0)$

***