W-7

ENTRANCE EXAMINATIONS – 2020

M.Sc. Statistics-OR

Hall Ticket No.	

Time : 2 hours
Max. Marks : 100

PART A: 50 Marks
PART B: 50 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.

2. Answers are to be marked on the OMR sheet following the instructions provided there upon.

3. Please read the instructions carefully before marking your answers on the OMR answer sheet.

4. Hand over the OMR answer sheet at the end of the examination to the Invigilator.

5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.

6. Calculators are not allowed.

7. There are a total of 50 questions in **PART-A** and **PART-B** together and each question carries **two marks.**

8. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. DO NOT USE A PENCIL.

9. This book contains **12 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.

10. Given below are the meanings of some symbols that may have appeared in the question paper: $\mathbb{R}$-The set of all real numbers, $E(X)$-Expected value of the random variable X, $V(X)$-Variance of the random variable X, $Cov(X,Y)$-Covariance of the random variables X and Y, $\rho_{X,Y}$ denotes the correlation coefficient between X and Y, iid-independent and identically distributed, pdf-probability density function, $B(n,p)$ and $N(\mu,\sigma^2)$ denote respectively,the Binomial and the Normal distributions with the said parameters. $Rank(A)$ and $det(B)$ mean rank and determinant of the matrices A and B respectively.

W-7

PART – A

1. The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 1-(1+x)e^{-x}, & x \geq 0. \\ 0, & \text{otherwise.} \end{cases}$$

Then Var(X) is

A. 4.
B. 2.
C. 6.
D. 1.

2. If $X \sim N(0,1)$, then $E(X^7)$ is

A. 1.
B. 0.
C. 9.
D. $\frac{1}{2}$.

3. If T is an unbiased estimator of θ, then

A. $E_\theta(T-\theta)^2 = Var(T)$.
B. $E_\theta(T-\theta)^2 \leq Var(T)$.
C. $E_\theta(T-\theta)^2 = \frac{1}{2}Var(T)$.
D. $E_\theta(T-\theta)^2 \geq Var(T)$.

4. A random variable X has moment generating function given by $M_x(t) = e^{5t+4t^2}$. Then the distribution of X is

A. Uniform distribution over (4,3).
B. Exponential distribution with parameter 3.
C. Normal distribution with mean 5 and variance 8.
D. Chi square distribution with 3 df.

5. Let X be a random variable with uniform distribution on $(0,1)$. Then $E(-2logX)$ is

A. $\frac{1}{2}$.
B. 2.
C. 4.
D. 1.

6. If X and Y are independent exponential random variable with same mean λ, then the distribution of $min(X, Y)$

A. Exponential with mean λ.

B. Exponential with mean 2λ.

C. Exponential with mean $\frac{\lambda}{2}$.

D. Exponential with mean $\frac{2}{\lambda}$.

7. Let X and Y be two independent Binomial random variables with parameters $(2, \frac{1}{3})$ and $(7, \frac{1}{3})$ respectively. Then $P[X + Y = 3]$ is equal to

A. $\binom{9}{3}(\frac{1}{3})^3(\frac{2}{3})^6$.

B. $\binom{9}{3}(\frac{2}{3})^3(\frac{1}{3})^6$.

C. $\binom{7}{2}(\frac{1}{3})^3 2(\frac{2}{3})^5$.

D. $\binom{9}{3}(\frac{1}{3})(\frac{2}{3})^8$.

8. Let T_1, T_2, T_3 be three treatments. Which of the following is not a contrast

A. $T_1 + 2T_2 - T_3$.

B. $T_1 - T_3$.

C. $T_1 + 2T_2 + T_3$.

D. $-T_1 + 2T_2 - T_3$.

9. In a completely randomized design with t treatments and n experimental units, error degrees of freedom is equal to

A. $n - t$.

B. $n - t + 1$.

C. $n - t - 1$.

D. $t - n$.

10. The ratio of two independent standard normal random variables has

A. a gamma distribution.

B. a beta distribution.

C. a standard cauchy distribution.

D. a normal distribution.

W-7

11. Empirical relation of averages is given by

A. Mean-Mode=2(Mean-Median).

B. Mean-Mode=3(Mean-Median).

C. Mean-Mode=2(Mean+Median).

D. Mean-Mode=3(Mean+Median).

12. Consider the following pairs

Concept	distribution
I. Mean=Variance	a. χ^2distribution
II. Mean¿Variance	b. Exponential(1)
III. 2Mean=Variance	c. Bionomial

A. I-b, II-c, III-a.

B. I-a, II-b, III-a.

C. I-b, II-a, III-c.

D. I-c, II-b, III-a.

13. To test the randomness of a sample, the appropriate test is

A. Run test.

B. Sign test.

C. Median test.

D. Mann-Whitney test.

14. Let n_1 and n_2 be integers with $n_1 \leq n_2$, and let a be a constant. Match the followings

Sequence	sum
I. $\sum_{i=n_1}^{n_2} a^i$	a. $\frac{1}{1-a}$, $\|a\| < 1$.
II. $\sum_{i=0}^{\infty} a^i$	b. $\frac{a_1^n}{1-a}$, $\|a\| < 1$.
III. $\sum_{i=1}^{\infty} a^i$	c. $\frac{a^{n_1} - a^{n_2+1}}{1-a}$, $a \neq 1$.
IV. $\sum_{i=n_1}^{\infty} a^i$	d. $\frac{a}{1-a}$, $\|a\| < 1$.

A. I-c, II-a, III-d, IV-b.

B. I-b, II-d, III-a, IV-c.

C. I-c, II-d, III-a, IV-b.

D. I-b, II-a, III-d, IV-c.

15. If X and Y are independent and Geometric distribution $G(\theta)$ then the conditional distribution $X|(X+Y)$ follows

A. Negative Binomial.

B. Poisson Distribution.

C. Binomial Distribution.

D. Discrete Uniform.

W-7

16. The average of n observations $x_1, ..., x_n$ is m. If x_1 is replaced by a, then the new average is

A. $m - x_1 + a$.

B. $\frac{m-x_1+a}{n}$.

C. $\frac{(n-1)m-x_1+a}{n}$.

D. $\frac{nm-x_1+a}{n}$.

17. If the data is contaminated with outliers then arrange following measures in according to least effect

I. Average

II. Median

III. Geometric mean

IV. Harmonic mean

A. II,IV,III,I.

B. II,IV,I,III.

C. II,III,IV,I.

D. III,II,IV,I.

18. Which of the following relations are true between Arithemetic mean (AM), Geometric Mean (GM) and Harmonic mean (HM)

I. $HM = \frac{(GM)^2}{AM}$.

II. $GM = \sqrt{AMXHM}$.

III. $AM \geq GM \geq HM$.

IV. $HM \geq AM \geq GM$.

A. Only I, II and III are true.

B. Only I, II and IV are true.

C. Only I, III and IV are true.

D. Only II, III and IV are true.

19. Arrange the steps invovling in fitting simple linear regression.

I. Compute correlation between the variables and generate scatter plot.

II. State assumptions.

III. Check for significance of regression coefficients.

IV. Validate the assumptions.

A. II-I-III-I.

B. IV-III-II-I.

C. II-III-IV-I.

D. III-II-IV-I.

W-7

20. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two head is at least 0.96 is

A. 6.

B. 8.

C. 5.

D. 9.

21. A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denote the probability that no two (or more) consecutive heads occurs then

I. p_1	a. $1-2p^2+p^3$
II. p_2	b. $1-p^2$
III. p_3	c. 1
IV. p_n	d. $(1-p)p_{n-1}+p(1-p)p_{n-2}$

A. I-c, II-b, III-a, IV-d.

B. I-b, II-a, III-d, IV-c.

C. I-c, II-d, III-a, IV-b.

D. I-b, II-a, III-d, IV-c.

22. A student has to match three historical events- Dandi March, Quit India Movement and Mahatma Gandhi's assassination with the years 1948, 1930 and 1942. The student has no knowledge of the correct answers and decides to match the events and years randomly. If X denotes the number of correct answer obtained by the student, then

I. $P(X=3)$	a. 1/2
II. $P(X=2)$	b. 0
III. $P(X=1)$	c. 1/6
IV. $P(X=0)$	d. 1/3

A. I-c, II-b, III-a, IV-d.

B. I-b, II-a, III-d, IV-c.

C. I-c, II-d, III-a, IV-b.

D. I-b, II-a, III-d, IV-c.

23. Suppose that $P(A)=3/5$ and $P(B)=2/3$. Then

A. $P(A\cup B)\geq 2/3$.

B. $4/15\leq P(A\cap B)\leq 3/5$.

C. $2/5\leq P(A|B)\leq 9/10$.

D. $P(A\cap B')\leq 1/3$.

W-7

24. Arrange the following random variables increasing order of their expected values:

(I.)$B(20, 1/4)$. (II.)$B(18, 1/3)$. (III.)$G(1/20)$. (IV.) $exp(20)$.

A. I-III-IV-II.

B. I-II-IV-III.

C. II-III-IV-I.

D. IV-I-II-III.

25. $X \sim P(7)$, that is X follows Poisson distribution with parameter 7, the correct arrangement of the following events in increasing order of their probabilities:

(I.)$\{X = 2\}$ (II.)$\{X = 5\}$ (III.)$\{X = 7\}$ (IV.)$\{X = 10\}$

A. I-III-II-IV.

B. I-IV-II-III.

C. I-III-II-IV.

D. IV-II-III-I.

PART - B

26. The negation of the statement "Some structures in Hyderabad were built over a 1000 years back" is

A. Some structures in Hyderabad were not built more than 1000 years ago.

B. No structure in Hyderabad was built over a 1000 years ago.

C. All structures in Hyderabad were built more than 1000 years ago.

D. At least one structure in Hyderabad was built over 1000 years ago.

27. Look at the columns below and match the entries regarding expected values and variances of random variables.:

I. $X \sim B(n,p)$	a. $E(X) = V(X)$
II. $X \sim exp(\lambda)$	b. $E(X) > V(X)$
III. $X \sim G(p)$	c. $E(X) < V(X)$
IV. $X \sim P(\lambda)$	d. Ordering of $E(X)$ and $V(X)$ depends on the parameter(s).

A. I-b, II-d, III-c, IV-a.

B. I-c, II-b, III-a, IV-d.

C. I-a, II-c, III-d,IV-b.

D. I-d, II-a, III-b, IV-c.

28. The absolute mean deviation is the least about

A. Minimum value.

B. Mean.

C. Median.

D. Maximum value.

W-7

29. Which of the following random variables has the largest median?

A. $U(-5, 5))$.

B. $U(-15, 5))$.

C. $U(-5, 15))$.

D. $U(-15, 15))$.

30. The median of 11 distinct positive numbers was reported to be 50, and the next number is 54 upon looking at the numbers again, it was realized that the number 74 was by mistake recorded as 47, after making this correction, the median will be

A. 54.

B. 47.

C. 74.

D. 50.

31. $X_1 \sim B(10, 1/5)$, $X_2 \sim B(10, 1/4)$, $X_3 \sim B(10, 1/2)$ $X_4 \sim B(10, 2/3)$.Identify the correct statement regarding their variances.

A. $V(X_4) > V(X_1) > V(X_3) > V(X_2)$.

B. $V(X_3) > V(X_1) > V(X_2) > V(X_4)$.

C. $V(X_1) > V(X_2) > V(X_3) > V(X_4)$.

D. $V(X_3) > V(X_4) > V(X_2) > V(X_1)$.

32. The correlation coefficient ρ_{XY} between random variables X and Y is 0.7, what is ρ_{UV}, where $U = 12 - 3X$ and $V = -13 + 4Y$?

A. 0.7.

B. 0.

C. 1.

D. -0.7.

33. $X_1, \ldots, X_n$ is a random sample from the $N(\mu, \sigma^2)$ population, an unbiased estimator for μ^2 is

A. $\bar{X}^2$.

B. $\frac{1}{n}\sum_{i=1}^{n} X_i^2$.

C. $\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$.

D. $\frac{1}{\binom{n}{2}} \sum_{1 \le i < j \le n} X_i X_j$.

W-7

34. Ashok does not know answers to any of the 50 multiple choice questions with 4 choices. He decides to randomly mark the options, a correct answer rewards 1 mark and a wrong answer awards $-\frac{1}{3}$, what are the expected marks that Ashok can get in this exam?

A. 25.

B. -16.667.

C. 0.

D. 50.

35. Look at the columns below and match the entries describing the random variables in the first column with the entries in the second column.

	Experiment	Random variable (rv)
I.	Number of red balls in a draw of 10 balls without replacing from a bag containing 12 red and 18 blue balls.	a. Binomial rv
II.	Number of red balls in a draw of 10 balls with replacement from a bag containing 12 red and 18 blue balls	b. Geometric rv
III.	The first time a red ball shows up in draws of one ball each with replacement from a bag containing 12 red and 18 blue balls	c. Hypergeometric rv

A. I-c, II-b, III-a.

B. I-b, II-a, III-c.

C. I-c, II-a, III-b.

D. I-a, II-b, III-c.

36. $0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 0$ are 12 independent observations from the random variable with probability mass function $Pr(X = x) = (\frac{\theta}{\theta+1})^x(\frac{1}{\theta})^{1-x}, x = 0, 1$, the maximum likelihood estimate for θ is

A. 5/12.

B. 7/12.

C. 5/7.

D. 4/7.

37. $X \sim exp(4)$, $\quad Pr(X \leq 15|X > 7)$ is equal to

A. $1 - e^{-32}$.

B. $1 - e^{-60}$.

C. e^{-32}.

D. e^{-28}.

W-7

38. To estimate the average monthly consumption of animal protein products - like fish, chicken, mutton etc. per household in a state, it was decided to first stratify the households and then draw samples of households from each stratum. A good criterion for stratification is

A. based on heights of the oldest person in the household.

B. based on distance of the households from the government secretariat.

C. based on the age of the youngest person in the household.

D. based on nature of employment of the best employed person in the household.

39. $X_1, \ldots, X_n$ is a random sample from the $N(\mu, \sigma^2)$ population. Let $\overline{X}$ be the sample mean and s^2 be the statistic $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ an unbiased estimator for μ^2 is

A. $\overline{X}^2$.

B. $\overline{X}^2 - 2s^2$.

C. $\overline{X}^2 - \frac{s^2}{n}$.

D. $(\overline{X} - s^2/n)^2$.

40. Let R_1, R_2 and R_3 be the rows of 3×3 non singular real matrix A. Let the first, second and third rows of the matrix B be $2R_1$, $R_2 + R_3$ and R_2, respectively. Then

A. B is singular.

B. rank(B)=3.

C. rank(A+B)=2.

D. rank(B-A)=2.

41. Consider a data set of marks in a public exam. 75% of marks in an exam are less or equal to 10. Using Chebsyeves inequality the standard deviation of the marks is

A. less than 4.

B. less than equal to 5.

C. equal to 4.

D. more than 5.

42. Let (X, Y) denote a randomly selected point in a square of area 1. What is the probability of the event that $|X - Y| \leq 1/3$?

A. 1/3.

B. 2/9.

C. 4/9.

D. 5/9.

W-7

43. X is a random variable with probability mass function $P(X = x) = pq^{x-1}, x = 1, 2, \ldots$, $0 < p < 1/2, p + q = 1$. Then

I. $P(X > j) = q^j$.
II. $E(X) > 1$.
III. $V(X) > E(X)$.
IV. $V(X) < E(X)$.

A. Only I, II, III are true.
B. Only I, II, IV are true.
C. Only I and IV are true.
D. None of the statements are true.

44. Let m_n, M_n and v_n be the mean, median and variance respectively of the first $n(\geq 5)$ natural numbers. Remove 1 and n from this set and let m_n', M_n' and v_n' denote the same for the new set. Then

I. $m_n' = m_n$, $v_n' < v_n$.
II. $M_n' = M_n$, $v_n' < v_n$.
III. $m_n' = m_n$, $M_n' = M_n$, $v_n' = v_n$.
IV. $m_n' < m_n$, $M_n' < M_n$.

A. Only I and II are true.
B. Only III and IV are true.
C. Only I and III are true.
D. Only II and IV are true.

45. 10 numbers are drawn from the set $\{1, 2, \ldots, 100\}$ randomly without replacement. The probability that

I. the mean of the selected numbers is more than 5.5 is greater than 0.99.
II. the median of the selected numbers is 6 is less than 0.0001.
III. the variance of the selected numbers is more than 8 is 1.
IV. the maximum of the selected numbers is more than 90 is greater than 0.5.

A. Only I is true.
B. Only II and IV are true.
C. Only II and III are true.
D. Only IV is true.

46. 6 girls $G_1, G_2, \ldots, G_6$ and 10 boys $B_1, B_2, \ldots, B_{10}$ are randomly made to sit in a row. What is the probability that none of the girls is at either end?

A. 1/4.
B. 3/8.
C. 1/2.
D. 5/8.

47. Match the followings

I. Cauchy Schwartz Inequality	a. $E(g(X)) \geq g(E(X))$, g-convex
II. Jensens's Inequality	b. $E(e^{tX}) \leq e^{t\mu}e^{t^2(b-a)^2/8}$ if $a < X < b$
III. Hoeffding's Inequality	c. $E\|XY\| \leq \sqrt{E(X^2)E(Y^2)}$, $Var(X) < \infty$, $Var(Y) < \infty$

A. I-c, II-a, III-b.

B. I-b, II-a, III-c.

C. I-a, II-c, III-b.

D. Incomplete options.

48. Match the followings

I. Gamma Distribution	a. Mean: 4, Variance: 2.4
II. Binomial distribution	b. Mean: 4, Variance: 4
III. Poisson distribution	c. Mean: 4, Variance: 6

A. I-a, II-b, III-c.

B. I-b, II-c, III-a.

C. I-c, II-a, III-b.

D. Incomplete options.

49. Order the following events in statistics software development in their chronological order

I. R-studio
II. S-Plus
III. R-shiny
IV. R

A. I-II-III-IV.

B. IV-III-II-I.

C. II-IV-I-III.

D. III-II-IV-I.

50. Order the following infinite series in increasing order of their limits

I. $1 + (2/3) + (4/9) + (8/27) + \ldots$
II. $-(1/2) + (1/4) - (1/8) + (1/16) - \ldots$
III. $(1/2) + (1/6) + (1/12) + (1/20) + \ldots$
IV. $1 - (1/2) + (1/3) - (1/4) + \ldots$

A. I-II-III-IV.

B. IV-III-II-I.

C. II-IV-III-I.

D. III-II-IV-I.