Entrance Examination – 2019 Ph.D. Statistics

Hall Ticket No.

Time : 2 hours Max. Marks : 70

PART A: 35 Marks PART B: 35 Marks

Instructions

- 1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Number in the space provided above.
- 2. Answers are to be marked on the OMR sheet.
- 3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
- 4. Hand over the OMR answer sheet at the end of the examination to the Invigilator.
- 5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
- 6. Calculators are not allowed.
- 7. There are a total of 40 questions in **PART A** and **PART B** together.
- 8. Each correct answer carries 1.75 marks and each wrong carries -0.50 marks.
- 9. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. DO NOT USE A PENCIL.
- 10. This book contains 7 pages including this page and excluding pages for the rough work. Please check that your paper has all the pages.
- 11. Given below are the meanings of some symbols that may have appeared in the question paper:

R-The set of all real numbers, E(X)-Expected value of the random variable X, V(X)-Variance of the random variable X, Cov(X, Y)-Covariance of the random variables X and $Y, \rho_{X,Y}$ denotes the correlation coefficient between X and Y, iid-independent and identically distributed, pdf-probability density function, B(n,p) and $N(\mu, \sigma^2)$ denote respectively, the Binomial and the Normal distributions with the said parameters. Rank(A) and det(B) mean rank and determinant of the matrices A and Brespectively.

- 1. Which of the following is not correct:
 - (A) If a distribution is skewed left, then the mean follows the tail and is less than the median.
 - (B) If a distribution is skewed right, then the mean is greater than the median.
 - (C) For a symmetric distribution, the mean and the median are equal.
 - (D) None of the above three statements are true.
- 2. What are the units of the standardised z-score?
 - (A) Same as the original data unit.
 - (B) Square root of the original data unit.
 - (C) Unitless.
 - (D) Square of the original data unit.
- 3. Roll two four-sided dice. With the numbers 1 through 4 on each die, the value of the roll is the number on the side facing downward. Assuming all 16 outcomes are equally likely. Which of the followings is not true.
 - (A) P[sum is at least 5]=5/8.
 - (B) P[sum is at least 5]=3/8.
 - (C) P[first die is 2]=1/4
 - (D) P[sum is at least 5 | first die is 2]=1/2
- 4. For Z a standard normal random variable, $E(Z^4)$ is equal to
 - (A) 0. (B) 1. (C) 2. (D) 3.
- 5. If the rank of 3×3 real matrix A is 3, it means that
 - (A) the determinant of A is zero.
 - (B) one of the rows of A is a linear combination of other rows.
 - (C) one of the columns of A is a linear combination of other columns.
 - (D) the rows of A form a basis in \mathbb{R}^3 .
- 6. The sum of three numbers is equal to 4, then their product is
 - (A) at least 4. (B) less than 3.
 - (C) equal to 3. (D) equal to 4.
- 7. 6 girls G_1, G_2, \ldots, G_6 and 10 boys B_1, B_2, \ldots, B_{10} are randomly made to sit in a row. What is the probability that none of the girls is at either end?

(A)
$$1/4$$
 (B) $3/8$ (C) $1/2$ (D) $5/8$

8. The series $\sum_{n=1}^{\infty} \frac{10^n}{n!}$.

(A) diverges. (B) equal to a positive number greater than 20.

(C) equal to 3. (D) equal to 10.

9. Given two sequences v_1, \ldots, v_n and w_1, \ldots, w_n , Cauchy-Schwarz inequality states that

(A)
$$(\sum_{i=1}^{n} v_i w_i)^2 \leq \sum_{i=1}^{n} v_i^2 \sum_{i=1}^{n} w_i^2$$

(B) $(\sum_{i=1}^{n} v_i w_i)^2 \leq (\sum_{i=1}^{n} v_i)^2 (\sum_{i=1}^{n} w_i)^2$
(C) $(\sum_{i=1}^{n} v_i w_i)^2 \geq (\sum_{i=1}^{n} v_i)^2 (\sum_{i=1}^{n} w_i)^2$
(D) $\sum_{i=1}^{n} v_i w_i \geq \sum_{i=1}^{n} v_i \sum_{i=1}^{n} w_i$

- 10. Let (X, Y) denote a randomly selected point in a square of area 1. What is the probability of the event that $|X Y| \le 1/3$?
 - (A) 1/3. (B) 2/9.
 - (C) 4/9. (D) 5/9.
- 11. What is the probability that the position in which the consonants appear remain unchanged when the letters of the word Math are re-arranged?

(A)
$$1/4$$
. (B) $1/6$. (C) $1/3$. (D) $1/24$.

- 12. How many five digit numbers can be formed using digits 0,1,2,3,4,5, Which are divisible by 3, without any of the digits repeating?
 - (A) 96. (B) 120. (C) 181. (D) 216.

13. What is the infinite sum
$$x + 2x^2 + 3x^3 + \dots$$
 for $|x| < 1$?

(A)
$$x/(1-x^2)$$
. (B) $x/(1+x)^2$. (C) $1/(1-x)^2$. (D) $x/(1-x)^2$.

14. If $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A is (A) $\begin{bmatrix} 3 & -4 \\ 3/4 & -1 \end{bmatrix}$. (B) $\begin{bmatrix} -13/4 & 3/2 \\ 5/4 & -1/2 \end{bmatrix}$. (C) $\begin{bmatrix} -17/4 & 3/4 \\ -7/4 & -1/4 \end{bmatrix}$. (D) $\begin{bmatrix} 5/4 & 11/4 \\ 3 & -9/4 \end{bmatrix}$.

15. Let a, b and also a - b, be unit vectors. Then what is the angle between a and b?

- (A) 60 degrees. (B) 120 degrees.
- (C) 90 degrees. (D) None of the above.
- 16. Let X be the number of heads on three independent flips of a biased coin that turns ups heads with probability p. The cumulative distribution function for X for $0 \le x < 1$ is

(A) 0. (B)
$$p^3$$
. (C) $1 - p^3$. (D) $(1 - p)^3$

- 17. Let f(x) be the pdf of a continuous random variable defined on \mathbb{R} ,
 - (A) f is an increasing function on \mathbb{R} .
 - (B) f is a continuous function.
 - (C) f > 0 for all x.
 - (D) f is always bounded.

- 18. The expected value of a random variable X taking positive integer values is 2, further P(X = 1) = 0.1, P(X = 2) = 0.2, P(X = 3) = 0.3, we can say that
 - (A) P(X = 4) can be equal to 0.16.
 - (B) P(X = 4) is less than 0.16.
 - (C) P(X = 5) can be equal to 0.2.
 - (D) P(X = 5) is more than 0.16.
- 19. the heights of adult ladies in our country are normally distributed with mean 150cm and variance 49cm^2 , if $P(0 < Z \le a) = 0.25$ where $Z \sim N(0, 1)$, then half the ladies have heights between
 - (A) (150 a) cm and (150 + a) cm.
 - (B) 150cm and (150 + 2a)cm.
 - (C) (150 7a) cm and (150 + 7a) cm.
 - (D) (150 7a) cm and (150 + a) cm.
- 20. $X \sim B(60, \frac{1}{3}))$. So
 - (A) $P(X = 48) \le P(X = 45)$.
 - (C) $P(X = 6) \le P(X = 54)$.
- (B) $P(X = 20) \le P(X = 48)$. (D) $P(X = 25) \le P(X = 30)$.

Part-B

- 21. In a simple linear regression situations, the circumstances dictate that the line contain the origin (i.e. $\alpha = 0$). In that case the least squares estimate of the slope (i.e. β) of the regression line is:
 - (A) $\hat{\beta} = (\sum_{i=1}^{n} x_i y_i) / (\sum_{i=1}^{n} x_i)^2$
 - (B) $\hat{\beta} = (\sum_{i=1}^{n} x_i y_i) / (\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i)$
 - (C) $\hat{\beta} = (\sum_{i=1}^{n} x_i y_i) / (\sum_{i=1}^{n} x_i^2)$
 - (D) None of the above three statements are true.
- 22. For simple linear regression the fitted value \hat{y} and residual $y \hat{y}$ their relationship is
 - (A) positively correlated. (B) negatively correlated.
 - (C) uncorrelated. (D) can't say.
- 23. The moment generating function for an exponential random variable is

(A) $\lambda/(\lambda - t)$. (B) $1/(\lambda - t)$. (C) $\lambda/(t - \lambda)$. (D) λ/t .

24. Let X be Exp(λ). Then exp(X) has which distribution?
(A) Gamma.
(B) Pareto.
(C) Log-Normal.
(D) Exponential.

25. Let Y and Z be two random variables with E(Z) = 0. Then Cov(Y, Z) =(A) 0. (B) E(Y). (C) E(YZ). (D) Can't say.

26. For m independent, α_I level hypothesis tests, show that the familywise error is

(A)
$$\alpha = 1 - ((1 - \alpha_I)^m).$$
 (B) $\alpha = 1 - (\alpha_I^m)$
(C) $\alpha = (1 - \alpha_I)^m.$ (D) $\alpha = \alpha_I^m.$

27. The CDF of Maximum order statistics from a sample of size n from $Uniform(0, \theta)$ distribution is given by:

(A)
$$(1 - (x/\theta))^n$$
.
(B) $(x/\theta)^n$.
(C) $x - (1/\theta)^n$.
(D) $1 - (x/\theta)^n$.

- 28. Marks on a Chemistry test follow a normal distribution with a mean of 65 and a standard deviation of 12. Approximately what percentage of the students have scores below 50?
 - (A) 11%
 - **(B)** 89%
 - (C) 15%
 - (D) 39%
- 29. Which of the following is not correct about a standard normal distribution?
 - (A) $P(Z \le -1.0) = 0.1587$
 - (B) $P(Z \ge 2.0) = 0.0228$
 - (C) $P(Z \le 1.5) = 0.9332$
 - (D) $P(Z \ge -2.5) = 0.4938$

- 30. Which of the following is not true assumptions of the Binomial distribution?
 - (A) All trials must be identical.
 - (B) Each trial must be classified as a success or a failure.
 - (C) The number of successes in the trials is counted.
 - (D) The probability of success is equal to 0.5 in all trials.
- 31. It is sometimes possible to obtain approximate probabilities associated with values of a random variable by using the probability distribution of a different random variable. For example, binomial probabilities using the Poisson probability function, binomial probabilities using the normal etc. In order for the Poisson to give good approximate values for binomial probabilities we must have the condition(s) that:
 - (A) the population size is large relative to the sample size.
 - (B) the sample size is large
 - (C) the probability, p, is small and the sample size is large
 - (D) the probability, p, is close to .5 and the sample size is large
- 32. X_1, \ldots, X_n are i.i.d. random variables with absolutely continuous distribution function $F(x;\theta)$, then $-\sum_{i=1}^n log F(X_i;\theta)$ has
 - (A) Normal distribution. (B) Beta distribution.
 - (C) Gamma distribution. (D) Weibull distribution.
- 33. The Central Limit Theorem states that
 - (A) if n is large, and if the population is normal, then the sampling distribution of the sample mean can be approximated closely by a normal curve.
 - (B) if n is large, and if the population is normal, then the variance of the sample mean must be small.
 - (C) if n is large, then the sampling distribution of the sample mean can be approximated closely by a normal curve.
 - (D) if n is large then the distribution of the sample can be approximated closely by a normal curve.
- 34. Which of the following statements is true. I. The standard error is computed solely from sample attributes. II. The standard deviation is computed solely from sample attributes. III. The standard error is a measure of central tendency.
 - (A) I only
 - (B) II only
 - (C) II and III only
 - (D) None of the above

1-60

- 35. T_1 and T_2 are two unbiased estimators of a parameter θ , however, $V(T_1) < V(T_2)$. So to estimate θ we should use the
 - (A) mean of the observed values of T_1 and T_2 because we will be using more information then.
 - (B) observed values of T_1 and T_2 by random selection to be unbiased.
 - (C) observed value of T_1 because it will be certainly closer to θ than the observed value of T_2 .
 - (D) observed value of T_1 because it is more likely to be closer to θ than the observed value of T_2 .
- 36. What will help ensure that the effect of a treatment is not due to some characteristic of a single experimental unit?
 - (A) Blinding. (B) Randomization. (C) Blocking. (D) Replication.
- 37. In a BIBD with t treatments in b blocks of k plots each and r replicates, which one of the following is not true?

(A) rt = bk. (B) $b \ge t$. (C) r > k. (D) $b \le (r + t - k)$.

- 38. Regarding a simple random sample of size n without replacement from a population of size N, identify the wrong statement
 - (A) Sample raw moments are unbiased estimators of the corresponding raw moments of the population.
 - (B) Every collection of n population units is equally likely to be the selected sample.
 - (C) The second central moment of the sample is not an unbiased estimator of the second central moment of the population.
 - (D) Every unit of the population is equally likely to be in the selected sample.
- 39. In a Markov chain with state space $\{0, 1, 2\}$ and one step transition matrix given by $\begin{bmatrix} 1/4 & 3/4 & 0 \end{bmatrix}$

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1/4 & 3/4 \end{bmatrix}, \text{ the value of } p_{01}^{(2)} \text{ will be:}$$
(A) 3/4. (B)9/16. (C) 3/16. (D) 7/16.

40. In a χ^2 test of independence, with m rows and n columns in the contingency table, the number of degrees of freedom associated with the test statistic is

(A) mn-1. (B) mn+1. (C) mn-m-n+1. (D) mn-m-n-1.

University of Hyderabad

Entrance Examinations - 2019 (June)

School/Department/Centre Course/Subject

: Mathematics and Statistics : Phd in Statistics

Q.No.	Answer	Q.No.	Answer	Q.No.	Answer	Q.No.	Answer
1	D	26	A	51	-NA-	76	-NA-
2	С	27	В	52	-NA-	77	-NA-
3	В	28	A	53	-NA-	78	-NA-
4	D	29	Ð	54	-NA-	79	-NA-
5	D	30	D	55	-NA-	80	-NA-
6	В	31	с	56	-NA-	81	-NA-
7	В	32	С	57	-NA-	82	-NA-
8	B	33	C	58	-NA-	83	-NA-
9	Α	34	А	59	-NA-	84	-NA-
10	D	35	D	60	'-NA-	85	-NA-
11	A	36	D	61	-NA-	86	-NA-
12	D	37	В	62	-NA-	87	-NA-
13 .	D	38	A	63	-NA-	88	-NA-
14	В	39	D	64	-NA-	89	-NA-
15	A	40	С	65	-NA-	90	-NA-
16	D	41	-NA-	66	-NA-	91	-NA-
17	B	42	-NA-	67	-NA-	92	-NA-
18	В	43	-NA-	68	-NA-	93	-NA-
19	с	44	-NA-	69	-NA-	94	-NA-
20	A	45	-NA-	70	-NA-	95	-NA-
21	с	46	-NA-	71	-NA-	96	-NA-
22	c	47	-NA-	72	-NA-	97	-NA-
23	Α	48	-NA-	73	-NA-	98	-NA-
24	В	49	-NA-	74	-NA-	99	-NA-
25	С	50	-NA-	75	-NA-	100	-NA-

Note/Remarks : 1. Total 40 questions only.

2. Questions 1-20 pertain to Part-A.

3. Questions 21-40 pertain to Part-B.

Hedreliz Bhalf 3. May - 19. (Faculty Coordinator, Admissions, PhD in Statistics) 305) DEAN 1/C________ DEAN 1/C_______