Entrance Examination: Ph.D. Statistics, 2017

Hall Ticket Number | | | | | | | | |

Time: 2 hours
Max. Marks: 80

Part A:40 Marks
Part B:40 Marks

Instructions

1. Write your Hall Ticket Number in the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 40 questions in **PART A** and **PART B** together.
8. Each correct answer carries 2 marks.
9. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
10. Given below are the meanings of some symbols that may have appeared in the question paper:
    $\mathbb{R}$-The set of all real numbers,$E(X)$-Expected value of the random variable X, $V(X)$-Variance of the random variable X, $Cov.(X,Y)$-Covariance of the random variables X and Y, $\rho_{X,Y}$ denotes the correlation coefficient between X and Y, iid-independent and identically distributed, pdf-probability density function, $B(n,p)$ and $N(\mu,\sigma^2)$ denote respectively,the Binomial and the Normal distributions with the said parameters.$Rank(A)$ and $det(B)$ mean rank and determinant of the matrices A and B respectively.
11. This book contains **8** pages including this page and excluding pages for the rough work. Please check that your paper has all the pages.

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1. Which among the following is the one out?

   - Dog.
   - Cat.
   - Ant.
   - Elephant.

   (A) Dog. **(B)** Cat. **(C)** Ant. **(D)** Elephant.

2. Which of the items given below fits with the following collection?

   Pen, Pendrive, Writing pad, Laptop, Pencil, Scale

   (A) Needle and thread. **(B)** Gas Lighter. **(C)** Elephant. **(D)** Eraser.

3. A and B are brothers, C is A's sister, D is the father of C, G is the mother of D, what is the relation of C to G?

   (A) C is a daughter of the lady G.

   (B) C is a granddaughter of the lady G.

   (C) C is a grandson of the lady G.

   (D) C is a son of the man G.

4. What is the next number in the sequence below?

   1, 5, 14, 30, 55, ...

   (A) 71. **(B)** 81. **(C)** 91. **(D)** 101.

5. The highest power of 10 that is a factor of 15! is
   (A) 3. **(B)** 6. **(C)** 5. **(D)** 2.

6. What should x be in the sequence given below:

   1 6 15 x 15 6 1

   (A) 11. **(B)** 20. **(C)** 6. **(D)** 10.

7. The difference between the squares of two consecutive odd numbers is 56, the product of the two numbers is
   (A) 105. **(B)** 143. **(C)** 195. **(D)** 255.

8. $A = \{a_1, a_2, a_3, a_4\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$, how many one-one functions f exist from A to B?
   (A) 20. **(B)** 60. **(C)** 100. **(D)** 120.

9. $B = \{1, 2, \ldots, N\} = \bigcup_{j=1}^{10} A_j$, each A_j is of the same size-say n, further every element of B belongs to exactly 6 of the A_js and every pair of elements of B belongs to exactly 3 of the A_js. What are N and n?
   (A) $N = 5, n = 3$. **(B)** $N = 6, n = 4$.
   (C) $N = 5, n = 2$. **(D)** N and n can not be determined from the given information.

10. In how many ways can the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ be arranged in a row so that neither any 2 even numbers, nor any 2 odd numbers are next to each other?
(A) $10!$. (B) $5!$. (C) $5! \times 5!$. (D) $2 \times 5! \times 5!$.

11. Ashok starts running from UoH to Gachhi Bowli stadium, while Bharat from Gachhi Bowli stadium to UoH which are $5km.$ apart at the same time on the same route, Ashok's speed is $1km./5min.$, while Bharat's speed is $1km./8min.$ and they both maintain the same speed throughout the distance. How much more will Bharat have to cover after crossing Ashok?

(A) Less than $2km.$.
(B) More than 2 but less than $3km.$.
(C) More than 3 but less than $4km.$.
(D) More than $4km.$

12. The diameter of a circle is increased by 10%, the ratio of percentage increases in the area and perimeter is
(A) 2.2. (B) 2.1. (C) 2. (D) 1.5.

13. A man borrows an amount on simple interest of 8% per year for 5 years and invests that amount on compound interest at 6%, after how many years will the difference between what he earns from his investment and what he returns will be positive?
(A) 5. (B) 4. (C) 6. (D) 2.

14. To make coffee for a party, Ashok mixed 1.5 cups of milk with 4 cups of water in a container, to this Bharat by mistake added one cup of milk, what should be done to this mix so that Ashok can make coffee as per his method?

(A) Add two cups of water.
(B) Add one and one third cups of water.
(C) Add two and two third cups of water.
(D) Add three and one third cups of water.

15. For any two digit number ab with $a > b$

(A) neither of them is a prime number.
(B) both $ab + ba$ and $ab - ba$ may be prime numbers.
(C) at most one $ab + ba$ and $ab - ba$ may be a prime number.
(D) None of the above is correct for every such a and b.

16. In an exam 3 marks are given for a correct answer -1 for a wrong answer and no marks are awarded if a candidate does not attempt a question, a candidate attempted all the 50 questions and scored 98 marks, how many questions did this candidate answer incorrectly?
(A) 11. (B) 13. (C) 15. (D) 17.

17. The number of digits in the binary representation of 324 is
(A) 6. (B) 7. (C) 8. (D) 9.

18. Two bags A and B contain 5 balls each which are numbered $1, 2, 3, 4, 5$, two balls are drawn independently from each of them without replacement and every pair is equally likely to be selected from each of the bags. What is the probability that the sum of the numbers on the selected balls from the two bags is the same?
(A) 0.25. (B) 0.16. (C) 0.125. (D) 0.08.

19. The ages of 5 children of a family starting from the youngest to the eldest are a, b, c, d, e, the median age is 10 years and so is the average, the age of the eldest is three times that of the youngest.The sum of the ages of the second youngest and the second eldest is the same as the sum of the ages of the youngest and the eldest, so it is possible that
(**A**) $b = 4,\ d = 16.$ (**B**) $b = 7,\ d = 13.$
(**C**) $b = 8,\ d = 16.$ (**D**) $b = 7,\ d = 16.$

20. it is not true that Ashok got top grades in at least 3 subjects, it means that

(**A**) Ashok failed in 3 subjects.

(**B**) Ashok got low grades in at least 3 subjects.

(**C**) Ashok got top grade in 4 or more subjects.

(**D**) Ashok got top grades in less than 3 subjects.

Part - B

21. If A_1, A_2 and A_3 are events for which $0 < P(A_i) < 1, \quad i = 1,2,3$ and $P(A_1|A_3) = P(A_3), \quad P(A_2|A_3) = P(A_2)$, then

    (A) $P(A_1 \cap A_2|A_3) = P(A_1 \cap A_2)$.

    (B) $P(A_1|A_2) = P(A_1)$.

    (C) $P(A_3|A_1 \cap A_2) = P(A_3)$.

    (D) None of the above can be said.

22. For a random variable X, $P(X > k) = (\frac{3}{4})^k, \quad k = 1,2,\ldots$, the expected value of X
    (A) is 1. **(B)** is 2.
    (C) is 3. **(D)** does not exist.

23. $\{X_n\}_1^\infty$ is a sequence of *iid* Bernoulli random variables with parameter p,let

$$T_n = X_1 + 2X_2 + \ldots + nX_n = \sum_{j=1}^{n} jX_j, \quad n = 1,2,\ldots$$

    (A) $P(\frac{T_n - \frac{n(n+1)}{2}p}{\sqrt{\frac{n(n+1)(2n+1)p(1-p)}{6}}} \leq x) \longrightarrow \Phi(x)$ as $n \to \infty$.

    (B) $\frac{T_n - \frac{n(n+1)}{2}p}{\sqrt{\frac{n(n+1)(2n+1)p(1-p)}{6}}} \longrightarrow 0$ in probability measure but not with probability one.

    (C) $\frac{T_n}{n} \longrightarrow 0$ with probability one.

    (D) $P(\frac{T_n - np}{\sqrt{np(1-p))}} \leq x) \longrightarrow \Phi(x)$ as $n \to \infty$.

24. X_1 and X_2 are independent random variables that are degenerate at -1 and 1 respectively, that is their distribution functions respectively are

$$F_1(x) = \begin{cases} 0 & x < -1 \\ 1 & x \geq -1 \end{cases} \qquad F_2(x) = \begin{cases} 0 & x < 1 \\ 1 & x \geq 1 \end{cases}$$

    the characteristic function $\phi_Y(t)$ of $Y = X_1 + X_2$ is
    (A) $\phi_Y(t) = i, \quad \forall t.$ **(B)** $\phi_Y(t) = e^{it}$.
    (C) $\phi_Y(t) = e^{-it}$. **(D)** $\phi_Y(t) = 1, \quad \forall t.$

25. Let A_n be the event that the $(2n-1)^{th}$ and the $(2n)^{th}$ tosses of a fair coin show heads, what can you say about the probability of the event B that two consecutive heads will occur infinitely many times when this coin is tossed again and again?
    (A) $P(B) = 0$. **(B)** $P(B) = 1$. **(C)** $P(B) = 1/2$. **(D)** $P(B) = 1/4$.

26. Arrivals of customers to a mall are in accordance with a Poisson process at the rate of 100 per minute, an arriving customer to the mall goes to the clothes section with probability 5/8, to the food section with probability 2/8 and to the electronics section with probability 1/8, the expected number of customers that will visit the electronic section in a 12 hours period is
    (A) 9000. **(B)** 12000. **(C)** 36000. **(D)** 72000.

27. Consider a homogeneous Markov chain $\{X_n, n = 0, 1, \ldots\}$ with state space $\{0, 2, 4, 6\}$ and the one step Transition probability Matrix. Identify the correct feature of this chain among the alternatives given below

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 2/3 & 0 & 1/3 & 0 \\ 0 & 5/6 & 0 & 1/6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(A) irreducible and recurrent.

(B) $C_1 = \{2\}$, $C_2 = \{0, 4\}$, $C_3 = \{6\}$ are closed communicating classes.

(C) 0 and 6 are recurrent states.

(D) only 6 is a recurrent state.

28. A coin for which the probability of showing heads upon tossing is p was tossed 10 times and heads appeared 7 times. The maximum likelihood estimate of p in the interval $([1/3, 41/60]$ is

(A) 1/3. **(B)** 7/10. **(C)** 41/60. **(D)** 1/2.

29. $X_1, \ldots, X_n$ is a random sample from the Poisson random variable with mean λ, let $\bar{X} = \frac{1}{n}(X_1 + \ldots + X_n)$ be the sample mean, an unbiased estimator for λ^3 is

(A) $\bar{X}^3$.

(B) $\bar{X}(\bar{X} - 1)(\bar{X} - 2)$ and it may take a negative value.

(C) $\bar{X}(\bar{X} - \frac{1}{n})(\bar{X} - \frac{2}{n})$ and it may take a negative value.

(D) $(\bar{X} - 1)^3$ and it may take a negative value.

30. $X_1, \ldots, X_n$ is a random sample from the $U((-\theta, \theta])$, $\theta > 0$, let $X_{(r)}$, $r = 1, \ldots, n$ denote the $r^{th.}$ order statistics, identify the correct statement

(A) $(X_{(1)}, X_{(n)})$ is jointly sufficient and $X_{(n)}$ is the maximum likelihood estimator for θ.

(B) $(X_{(1)}, X_{(n)})$ is jointly sufficient and $\frac{1}{2}(X_{(n)} - X_{(1)})$ is the maximum likelihood estimator for θ.

(C) $X_{(n)}$ is a sufficient and is also the maximum likelihood estimator for θ.

(D) $(X_{(1)}, X_{(n)})$ is jointly sufficient and $\max(-X_{(1)}, X_{(n)})$ is the maximum likelihood estimator for θ.

31. A test of a given size based on one sample point X for $H_0 : f(x) = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}$, $-\infty < x < \infty$ vs. $H_1 : f(x) = \frac{1}{2}e^{-|x|}$, $-\infty < x < \infty$ is to reject H_0 if $|X - 1| > 1/2$, the power of this test is

(A) less than $1 - e^{-1/2}$.

(B) equal to $e - 1$.

(C) equal to $e^{-1/2} - e^{-3/2}$.

(D) equal to $1 - e^{-1/2} + e^{-3/2}$.

32. 3.1, 1.4, 2.4, 1.7, 1.9 is a random sample from the $U((\theta, \theta+2))$ population, the maximum likelihood estimate of θ based on this sample

    (A) either of 3.1 or 3.6.

    (B) is unique and is 3.1.

    (C) any value in the interval $(1.4, 3.1)$ maximizes the likelihood function.

    (D) any value in the interval $(1.1, 1.4)$ maximizes the likelihood function.

33. $\Phi(-1.96) = .025, \Phi(-1.64) = .05$, based on a random sample of size 16 from the $N(\mu, \sigma^2)$ population, the sample mean is 46 and the length of the 95% confidence interval is 12, σ is very close to

    (A) 11. **(B)** 9 . **(C)** 4. **(D)** 6.

34. X is a random variable whose second moment exists, the probability that it takes values less than 15 less than its mean or more than 15 more than its mean is 0.6,

    (A) $V(X) \leq 100$.

    (B) $V(X)$ could be 125.

    (C) $V(X)$ could be 144.

    (D) Nothing can be said about $V(X)$.

35. X_i, $i = 1, 2, 3$ are *iid* standard normal random variables, $E(2X_1+3X_2|X_1+3X_2-X_3 = 4)$ is

    (A) 4/11. **(B)** 4 . **(C)** 0. **(D)** 4/13.

36. w_1, w_2, w_3, w_4 are weights of 4 kinds of tennis balls, all balls of a particular kind weigh the same, a weighing balance was used and the following balances were achieved, you may assume that in each of these there are unobservable random errors - ϵ_i, $i = 1, 2, 3$ which are *iid* Normal random variables with mean 0 and variance σ^2

$$w_1 + w_2 = 10 + w_3 + \epsilon_1$$
$$w_2 + w_3 = 7 + w_1 + \epsilon_2$$
$$w_1 + w_3 = 8 + w_2 + \epsilon_2$$

    (A) The $BLUE$ of $4w_1 + 5w_2 - 3w_3$ does not exist.

    (B) The $BLUE$s of some linear combinations of w_1 and w_3 do not exist.

    (C) The $BLUE$s of every linear combination of w_1, w_2 and w_3 exist.

    (D) None of the above is correct.

37. In a large farm, there are N rows of mango trees, each row has M trees, a simple random sample of $n(< N)$ rows is selected and the simple random samples of $m(< M)$ trees are selected from each of the selected rows, the yields of each of the selected trees are observed, this sampling scheme is

    (A) a stratified sampling scheme and we can estimate the average yield per tree in the entire farm.

    (B) a single stage clustering sampling scheme and we can not estimate the average yield per tree in the entire farm.

    (C) a two stage clustering sampling scheme with distinct cluster sizes and we can estimate the average yield per tree in the entire farm.

(D) a two stage clustering sampling scheme with equal cluster sizes and we can estimate the average yield per tree in the entire farm.

38. Each of the seven treatments have to appear in blocks of sizes four each. Which of the following choices on "number of blocks" and "number of blocks in which a pair of treatment appear" respectively, gives a valid BIBD.
(A) $7, 2$. **(B)** $6, 3$. **(C)** $8, 3$. **(D)** $7, 1$.

39. In a 2^4 factorial design, the following combinations are in Block-1 of a replication, which is the effect that is confounded?

$$a \quad c \quad d \quad ab \quad bc \quad bd \quad acd \quad abcd$$

(A) $ABCB$. **(B)** ACD. **(C)** ABC. **(D)** BCD.

40. A is a $n \times n$ real matrix which has two complex eigen values and A^T is its transpose, what can you say about the eigen values of $B = A + A^T$?

(A) B has 2 complex eigen values.

(B) B has no complex eigen value.

(C) All the eigen values of B are non-negative.

(D) If $n > 3$, B will have 4 complex eigen values.