ENTRANCE EXAMINATION – 2016

QUESTION PAPER BOOKLET

Ph.D. (PHYSICS)

Marks: 75

Time: 2.00 hrs.

Hall Ticket No.:

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- I. Please enter you Hall Ticket Number on Page 1 of this question paper and on the OMR sheet without fail.
- **II.** Read carefully the following instructions:
 - 1. This Question paper has two Sections: Section A and Section B
 - Section A consists of 25 objective type questions of one mark each.
 There is negative marking of 0.33 mark for every wrong answer.
 The marks obtained by the candidate in this Section will be used for resolving the tie cases.
 - 3. Section B consists of 50 objective type questions of one mark each. There is no negative marking in this Section.
 - 4. Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below



- 5. Only Non programmable Scientific Calculators are permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
- 6. Hand over the OMR answer sheet at the end of the examination to the Invigilator.

This book contains 24 pages

III. Values of physical constants:

 $c = 3 \times 10^8$ m/s; $h = 6.63 \times 10^{-34}$ J.s; $k_B = 1.38 \times 10^{-23}$ J/K $e = 1.6 \times 10^{-19}$ C; $\mu_{\circ} = 4\pi \times 10^{-7}$ Henry/m; $\epsilon_{\circ} = 8.85 \times 10^{-12}$ Farad/m

SECTION A

1. The inverse of
$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$
, $b \neq a \neq 0$ is

(A)
$$\begin{pmatrix} a & +b \\ -b & a \end{pmatrix}$$

(B) $\frac{1}{a^2+b^2} \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$
(C) $\frac{1}{a^2+b^2} \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$
(D) $\frac{1}{a^2-b^2} \begin{pmatrix} -b & a \\ a & b \end{pmatrix}$

- 2. Which of the following is an appropriate Lagrangian for a charged particle (of charge q, mass m) in an electromagnetic field with scalar potential ϕ and vector potential \vec{A} ?
 - (A) $L = \frac{1}{2}m\dot{r}^2 q(\phi \vec{A} \cdot \vec{r})$ (B) $L = \frac{1}{2}m\dot{r}^2 - q(\phi + \vec{A} \cdot \vec{r})$ (C) $L = \frac{1}{2}m\dot{r}^2 + q(\phi - \vec{A} \cdot \vec{r})$ (D) $L = \frac{1}{2}m\dot{r}^2 + q(\phi + \vec{A} \cdot \vec{r})$

3. The integral $\oint_C \frac{dz}{z^2 - 1}$ where C is the contour such that |z| = 2 is

(A) 0 (B) πi (C) $\frac{\pi i}{2}$ (D) $\frac{\pi}{2}$ 4. $\int_{0}^{1} \frac{x}{1+x^{2}} dx =$ (A) 1 (B) $\pi/4$ (C) $\tan^{-1} \frac{\sqrt{2}}{2}$ (D) $\log \sqrt{2}$

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- 5. In a quantum mechanical Coulomb system, the energy levels are :
 - (A) equally spaced.
 - (B) with spacing that increases with n^2 .
 - (C) with spacing that decreases with n^2 .
 - (D) with spacing that decreases with n^3 .
- 6. If $L = \frac{1}{2}m\dot{r}^2 + \frac{e^2}{4m\epsilon_0 r}$ be the Lagrangian of a system (particle) under a central force force, then what are the conserved quantities associated with the motion of the particle?
 - (A) Energy and angular momentum
 - (B) Energy, angular momentum, and Laplace-Runge-Lenz vector
 - (C) Energy, angular momentum, and radial distance
 - (D) Energy, angular momentum, Laplace-Runge-Lenz vector and radial distance
- 7. A charged particle moving in non-zero perpendicular electric and magnetic fields follows a
 - (A) Circular path
 - (B) Elliptical path
 - (C) Helical path
 - (D) Straight line
- 8. A dipole of dipole moment \vec{P} is placed in the external \vec{E} . The force acting on the dipole is given by
 - (A) $-\vec{P}\times\vec{E}$
 - (B) $\nabla \times (\vec{P} \times \vec{E})$
 - (C) $-\vec{P}\cdot\vec{E}$
 - (D) $\vec{\nabla}(\vec{P}\cdot\vec{E})$
- 9. A student obtains a large amount of data which she has to numerically fit using one of the given methods. Which method would be suitable for fitting her data?
 - (A) Gauss Jordan Method
 - (B) Trapezoidal Method
 - (C) Lagrange's interpolation
 - (D) Newton-Raphson method

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- 10. An electron and a proton are injected into a uniform magnetic field perpendicular to it with the same momentum. Which of the following is true?
 - (A) Radius of curvature is less for electron
 - (B) Radius of curvature is more for the electron
 - (C) Radius of curvature is the same for both
 - (D) They both move in straight lines.
- 11. There are 4 particles in a one dimensional box of width L with hard walls. If they are fermions, the total ground state energy is E_F , if bosons, it is E_B and if they are classical particles they the energy is E_{cl} . Which of the following is true?
 - $(\mathbf{A}) \quad E_F = E_B = E_{cl}$
 - $(\mathbf{B}) \quad E_F > E_B = E_{cl}$
 - (C) $E_F < E_B < E_{cl}$
 - (D) $E_F > E_B > E_{cl}$
- 12. At the Brewster's angle the reflected light from the dielectric is polarized
 - (A) parallel to the plane of incidence.
 - (B) perpendicular to the plane of incidence.
 - (C) may have any arbitrary polarization.
 - (D) depends on the nature of the dielectric material.

13. What is the coordination number of FCC crystal?

- (A) 6
- **(B)** 8
- **(C)** 12
- **(D)** 16

14. The binary number 1011 0110 is equivalent to the decimal number

- (A) 180
- **(B)** 64
- (C) 182
- **(D)** 132

15. For operators A and B satisfying the relation $[\hat{A}, \hat{B}] = 1$. $[\hat{A}, \hat{B}^2]$ is given by

- (A) $\hat{A}\hat{B}$
- **(B)** \hat{B}
- (C) $2\hat{B}$
- (D) Â
- 16. At what value of the speed v does the de Broglie wavelength of a micro-particle equal its Compton wavelength?

. . .

- (A) $v = \frac{C}{\sqrt{2}}$
- (B) $v = \frac{C}{2}$
- (C) $v = C^{\perp}$

(D)
$$v = \frac{\sqrt{C}}{2}$$

17. In the nuclear process

$$_{e}C^{11} \longrightarrow _{5}B^{11} + e^{+} + X$$

X stands for

- (A) neutron
- (B) neutrino
- (C) antineutrino
- (D) photon
- 18. A spring of constant k is stretched a certain distance. It takes twice as much force to stretch a second spring to half the distance. The spring constant of the second spring is
 - (A) k
 - **(B)** 2k
 - (C) 4k
 - (D) 8k

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- 19. Isospin symmetry implies that
 - (A) electron and proton are indistinguishable.
 - (B) electron and neutron are indistinguishable.
 - (C) electron and photon are indistinguishable.
 - (D) proton and neutron are indistinguishable.
- 20. In beta decay which of the following invariances is violated?
 - (A) Gauge Invariance
 - (B) Translational Invariance
 - (C) Reflection Invariance
 - (D) Rotation Invariance
- 21. The electron energy states in a solid are shown in terms of bands. In a semiconductor we have electrons in the conduction band, inner bands and valance bands. Which of the following is correct?
 - (A) The upper most band is the valance band.
 - (B) All semiconductors are electrically neutral.
 - (C) With increase of temperature, the resistivity of a semiconductor decreases.
 - (D) Addition of a small amount of impurity III and IV group element to a pure semiconductor increases its resistivity.
- 22. A resistor in a circuit dissipates energy at the rate of 1 W. The voltage across the resistor is doubled, what will be the new rate of dissipation
 - (A) .25W
 - **(B)** 4W
 - (C) 2W
 - (\mathbf{D}) .5W
- 23. The root mean square velocity of molecules of mass m of an ideal gas at temperature T is
 - (A) 0
 - (B) $\sqrt{\frac{3kT}{m}}$
 - (C) $\sqrt{\frac{kT}{m}}$
 - (D) $\sqrt{\frac{3kT}{m\pi}}$

24. Which of these ions cannot be used as a dopant in germanium to make an n-type semi-conductor

. . . .

- (A) As
- **(B)** B
- (C) Sb
- (D) N
- 25. A pipe closed at one end and open at the other end has length L. If c_s is the velocity of sound, then the fundamental frequency of this closed pipe is
 - (A) $\frac{c_s}{2L}$
 - (B) $\frac{c_s}{4L}$
 - (C) $\frac{3c_s}{2L}$
 - (D) $\frac{3c_s}{2L}$

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SECTION-B

26. The action of a linear operator O on an element x(t) of the vector space of all polynomials in t is given by O x(t) = x(t-1). then $O^{-1}x(t)$ is given by

- (A) $x(\frac{1}{t}-1)$
- **(B)** x(t+1)
- (C) $x(\frac{1}{t}+1)$
- (D) $x(\frac{2}{t}-1)$

27. The solution of the second order differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 6$ is

- (A) $c_1 e^{-x} + c_2 e^{3x}$
- (B) $c_1 e^{-3x} + c_2 e^x$
- (C) $c_1 e^{-x} + c_2 e^{3x} 2$
- (D) $c_1e^{-x} + c_2e^{3x} + 6$

28. Which of the following are the eigenvalues of the matrix $\begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$

- (A) 1-i, 1+i
- **(B)** 1,3
- **(C)** 2,2
- (**D**) i,-i
- 29. A particle emerging from an accelerator has relativistic energy 10GeV and relativistic momentum $8\frac{GeV}{c}$. The rest mass of the particle is
 - (A) 0.25 $\frac{GeV}{c^2}$
 - (B) 1.20 $\frac{GeV}{c^2}$
 - (C) 2.00 $\frac{GeV}{c^2}$
 - (D) $6.00 \frac{GeV}{c^2}$

30. For $0 < t < \pi$, the matrix $\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$ has distinct complex eigenvalues λ_1 and λ_2 . For what value of $t, 0 < t < \pi$, is $\lambda_1 + \lambda_2 = 1$?

- (A) $\pi/6$
- (B) $\pi/4$
- (C) $\pi/3$
- (D) $\pi/2$

- 31. A particle is trapped in a one dimensional potential given by $V(x) = \frac{1}{2}kx^2$. At a time t = 0 the state of the particle is described by the wave function $\psi(x) = c_1\psi_0(x) + c_2\psi_1(x)$, where $\psi_n(x)$ is the eigenfunction belonging to the eigen value E_n . The expected value of energy $\langle E \rangle$ is
 - (A) $\frac{1}{2}\hbar\omega(c_1^2+3c_2^2)$
 - (B) $\frac{1}{2}\hbar\omega$
 - (C) $\frac{1}{2}\hbar\omega(c_1^2+2c_2^2)$
 - (D) $\frac{1}{2}\hbar\omega(c_1+2c_2)^2$
- 32. Motion of ripples of short wave length on water is controlled by surface tension S. Phase velocity of such ripples is given by $V_o = \left(\frac{2\pi S}{\rho\lambda}\right)^{1/2}$, where ρ is density of water. What is the group velocity for disturbance mode of wavelength close to λ ?
 - (A) V_p
 - (B) $\frac{3}{2}V_p$
 - (C) $\frac{1}{2}V_p$
 - (D) $\frac{3}{4}V_p$
- 33. Which of the following is an eigenstate of the Pauli matrix σ_x

$$(A) \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$(B) \quad \begin{pmatrix} 1 \\ -i \end{pmatrix}$$
$$(C) \quad \begin{pmatrix} 1 \\ i \end{pmatrix}$$
$$(D) \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- 34. The amplitude of the resultant wave obtained by adding $E_1 = E_o \sin \omega t$ and $E_2 = E_o \sin(\omega t + 60)$ is
 - (A) $E_r = \sqrt{3}E_o$
 - (B) $E_r = \sqrt{2}E_o$
 - (C) $E_r = \frac{3}{2}E_o$
 - (D) $E_r = 2E_o$

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- 35. Force per unit area between two parallel infinite current sheets with current densities λ_1 and λ_2 in the same direction is
 - (A) $\frac{\mu_0\lambda_1\lambda_2}{2}$
 - (B) $\frac{\mu_0\lambda_1\lambda_2}{4}$
 - (C) $4\mu_0\lambda_1\lambda_2$
 - (D) $2\mu_0\lambda_1\lambda_2$
- 36. The dipole moment of a thin charged rod bearing charge density $\rho = \lambda \delta(x) \delta(y)$ for $z \in (-a, a)$ is
 - (A) $\frac{2}{3}\lambda a^3 \hat{z}$
 - (B) $\frac{1}{12}\lambda a^3 \hat{z}$
 - (C) $\frac{1}{6}\lambda a^3 \hat{z}$
 - (D) $\frac{1}{3}\lambda a^3 \hat{z}$
- 37. The electric field of a plane electromagnetic wave propagating in free space along z-direction is given as $\vec{E}(z,t) = E_0 \cos(kz \omega t)\hat{x}$. The time averaged value of the Poynting vector $(\langle \vec{S} \rangle)$ for such wave is
 - (A) $\frac{1}{2}\epsilon_0 E_0^2 \hat{z}$
 - (B) $\frac{1}{2}\epsilon_0 E_0^2 \hat{z}$
 - (C) $\frac{1}{2\mu_0} c E_0^2 \hat{z}$
 - (D) $\frac{1}{2}c\epsilon_0 E_0^2 \hat{z}$
- 38. Two linearly polarized light waves of same frequency and amplitude are superimposed such that their polarizations are perpendicular to each other. What should be phase difference between these two waves to generate circularly polarized light
 - (A) π
 - **(B)** $\pi/2$
 - (C) $\pi/4$
 - (D) $\pi/3$
- 39. The solution to the Schrödinger equation for a particle in an infinitely deep potential well indicates that in the middle of the well, the probability density vanishes for
 - (A) ground state (n = 0) only
 - (B) state of even $n(n = 2, 4, 6, \cdots)$
 - (C) states of odd $n(=1,3,5,\cdots)$
 - (D) all states $(n = 1, 2, 3, \cdots)$

- 40. The probability of occupation of a quantum state with energy 0.10 eV above the Fermi energy to the occupied at temperature 800°K (with $k = 8.62 \times 10^{-5} \text{ eV/K}$) is
 - (A) 10%
 - **(B)** 81%
 - (C) 19%
 - **(D)** 90%

41. For the relativistic decay $A \longrightarrow B + C$, with A at rest, the energy of B is

- (A) $\frac{M_A^2 + M_B^2 + M_C^2}{2M_A}$ (B) $\frac{M_A^2 + M_B^2 - M_C^2}{2M_A}$ (C) $\frac{M_A^2 + M_C^2 - M_B^2}{2M_B}$ (D) $\frac{M_A^2 + M_B^2 - M_C^2}{2M_C}$
- 42. A nucleus of mass M emits a γ -ray photon of frequency ν . What is the loss of internal energy by the nucleus?
 - (A) $h\nu$
 - (B) $h^2 \nu^2 / 2Mc^2$
 - (C) $h^2\nu(1+h\nu/2Mc^2)$
 - (D) $h\nu(1-h\nu/2Mc^2)^*$
- 43. In a Frank-Hertz type experiment, atomic hydrogen is bombarded with electron and excitation potentials are found at 10.21 V and 12.10 V. The wavelengths of different lines of spectral emission accompanying these excitations are?
 - (A) 121.5 nm, 102.5 nm and 656.7 nm
 - (B) 121.5 nm, 102.5 nm
 - (C) 121.5 nm, 102.5 nm and 242 nm
 - (D) 121.5 nm, 102.5 nm and 205 nm

- 44. According to BCS theory the attraction between the electrons of a Cooper pair is due to
 - (A) weak force
 - (B) vacuum polarization
 - (C) Casimir force
 - (D) exchange of quanta of lattice vibrations.
- 45. The primitive translation vectors of a two-dimensional lattice are

$$\vec{a} = 2\hat{i} + \hat{j}$$
 and $\vec{b} = 2\hat{j}$

assume that the third vector and of the lattice is along z-axis with unit magnitude $(\vec{c} = \hat{k})$. The primitive translation vectors of its receptocal lattice is

- (A) $\vec{a}^* = \pi \hat{i}$ and $\vec{b}^* = \frac{\pi}{2}(-\hat{i}+2\hat{j})$
- (B) $\vec{a}^* = 2\pi \hat{i}$ and $\vec{b}^* = \pi (\hat{i} + \hat{j})$
- (C) $\vec{a}^* = \pi(\hat{i} + \hat{j})$ and $\vec{b}^* = \pi(\hat{i} \hat{j})$
- (D) $\vec{a}^* = \pi(\hat{i} \hat{j})$ and $\vec{b}^* = \pi(\hat{i} + \hat{j})$
- 46. A metal has free electron density $2 \times 10^{28}/m^3$. The charge relaxation time is given by 7.1×10^{-14} sec. The static conductivity of the metal is given by
 - (A) 4×10^7 mho/m
 - (B) 8×10^7 mho/m
 - (C) 15×10^7 mho/m
 - (D) 30×10^7 mho/m
- 47. The total parity of any quantum mechanical state ψ with intrinsic parity η_{ψ} and orbital angular momentum l is given by
 - (A) $\eta_{TOT} = \eta_{\psi}(-1)^l$
 - (B $\eta_{TOT} = \eta_{\psi}(-1)^{l+1}$
 - (C) $\eta_{TOT} = (\eta_{\psi} + 1)$
 - (**D**) $\eta_{TOT} = (\eta_{\psi} 1)^{l}$

48. The angular resolution of a telescope is given by

- (A) $\frac{2.44\lambda}{D}$
- (D) $\frac{1.22D}{2}$

- 49. The mediator W of the weak interaction has a mass $M = 80 \text{ Gev/c}^2$. In the framework of a Yukawa like model, the range of the weak interaction is
 - (A) 2.5×10^{-15} m.
 - (B) 2.5×10^{-18} m.
 - (C) 2.5×10^{-15} m.
 - (D) infinite.
- 50. If h be the step size the local truncation error for the fourth order Runge Kutta Method is of the order of
 - (A) h^4
 - **(B)** h^5
 - (**C**) h
 - (D) h^2
- 51. A committee has 6 ladies and 5 gentlemen. A 5-member subcommittee is to be selected at random from these 11 committee members. Find the probability that the committee has 2 ladies and 3 gentlemen
 - (A) 200/462
 - **(B)** 81/462
 - (C) 150/462
 - **(D)** 240/462
- 52. A gas consists of non-interacting and identical particles, whose energy depends on its momentum in a non-trivial way. $\epsilon = C|p|^n$. The energy per particle in the gas is found to be [you may use $\int dx x^{\alpha-1} e^{-x} = \Gamma[\alpha]$]
 - (A) 3T/2
 - (B) $\frac{3}{n}T$
 - (C) $\frac{2}{3}T$
 - (D) $\frac{2}{n}T$

53. Molecular oxygen has a net magnetic spin \vec{s} unity, i.e. s_z is quantized to -1, 0, +1. The partition function z of such a system in a magnetic field $\vec{B} || \hat{z}$ is given by

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$$z = \frac{1}{N!} \left(\frac{v}{\lambda^3} (e^{\beta \mu B} + 1 + e^{-\beta \mu B})^N \right)$$

The average magnetic dipole moment is given by

- (A) $N\mu\left(\frac{2\sinh\beta\mu B}{2\cosh\beta\mu B+1}\right)$ (B) $N\mu\left(\frac{2\cosh\beta\mu B}{2\cosh\beta\mu B+1}\right)$ (C) $N\mu\left(\frac{2\sinh\beta\mu B}{2\sinh\beta\mu B+1}\right)$ (D) $N\mu\left(\frac{2\cosh\beta\mu B}{2\sinh\beta\mu B+1}\right)$
- 54. Which of the following relations (with the eigenstate of angular momentum operator) is most appropriate
 - (A) $0 \leq \langle \psi_{l,m} | (\hat{L}_x^2 + \hat{L}_y^2) | \psi_{l,m} \rangle \leq \hbar^2 (l(l+1) m^2)$
 - (B) $0 \le \langle \psi_{l,m} | (\hat{L}_x^2 + \hat{L}_y^2) | \psi_{l,m} \rangle \le (l(l+1)\hbar^2)$
 - (C) $m^2\hbar^2 \leq \langle \psi_{l,m} | (\hat{L}_x^2 + \hat{L}_y^2) | \psi_{l,m} \rangle \leq (l(l+1))$
 - (D) $l\hbar^2 \leq \langle \psi_{l,m} | (\hat{L}_x^2 + \hat{L}_y^2) | \psi_{l,m} \rangle \leq (l(l+1))$
- 55. In a one step random walk, you flip a coin, if it is heads you take a step, if it is tail, you do not take a step. If all your stepsizes are equal (1 cm), for a walk of 10 steps what is your probability of reaching any point less than 10 cms?
 - (A) 1/10
 - **(B)** 1/2
 - (C) 2/10
 - **(D)** 2/5
- 56. Suppose that we measure the acceleration due to gravity g, using a simple pendulum. The period of such pendulum is given by $T = 2\pi \sqrt{\frac{l}{g}}$, where l is the length of the pendulum and T is the time period. In a particluar measurement, the errors in measuring l and T are 0.1% and 0.2% respectively. The percentage error in measuring g is given by
 - **(A)** 0.1%.
 - **(B)** 1%.
 - (C) 0.3%.
 - **(D)** 2%

- 57. The invariant mass spectrum of Λ° and π^{+} in the reaction $K^{0}+p \rightarrow \Lambda^{\circ}+\pi^{+}+\pi^{-}$ shows a peak Y_{1}^{*} , at 1385 MeV with a full width of 50 MeV. The strangeness, hypercharge and isospin of Y_{1}^{*} are
 - (A) S = -1, Y = 0, I = 1
 - (B) S = 0, Y = 0, I = 1
 - (C) S = -1, Y = 0, I = 0
 - (D) S = 0, Y = 1, I = 0
- 58. In a certain colliding-beam storage ring, protons of energy 30 GeV collide head on. The energy that a single proton must have to give the same centre-of-mass energy when colliding with a stationary proton is
 - (A) 1 GeV
 - **(B)** 30 GeV
 - (C) 900 GeV
 - (**D**) 1800 GeV

59. The decay $\mu^- \rightarrow e^- + \gamma$ is not observed because

- (A) Baryon number is not conserved.
- (B) Strangeness is not conserved.
- (C) Lepton number is not conserved.
- (D) Isospin is not conserved.
- 60. The probability for an event occuring is given by the Poisson distribution defined by $W(n) = \frac{\lambda^n}{n!} e^{-\lambda}$. If λ is the mean number of events, the variance of the distribution is given by
 - (A) λ
 - (B) $\lambda/2$
 - **(C)** 2λ
 - **(D)** 0

61. If v is the velocity of a galaxy and r is its distance, then according to Hubble's law

- (A) $v \propto r^3$
- (B) $v \propto \frac{1}{r}$
- (C) $v \propto r$
- (D) $v \propto \frac{1}{r^2}$

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- 62. The cutoff wavelength of TE_{10} mode in a rectangular wave guide of dimensions a and b with a > b is

- (A) a/2
- **(B)** 2a
- (C) $\pi(b/2)$
- (D) $\sqrt{a^2+b^2}/2$
- 63. If a uniform electric field is directed along z-direction the diagonal components of Maxwell's stress tensor will be
 - (A) $T_{xx} = 0, T_{yy} = 0, T_{zz} = 0.$
 - (B) $T_{xx} = 0, T_{yy} = 0, T_{zz} = \frac{1}{2}\epsilon_0 E_0^2$
 - (C) $T_{xx} = -\frac{1}{2}\epsilon_0 E_0^2, T_{yy} = -\frac{1}{2}\epsilon_0 E_0^2, T_{zz} = \frac{1}{2}\epsilon_0 E_0^2$
 - (D) $T_{xx} = \frac{1}{2}\epsilon_0 E_0^2, T_{yy} = \frac{1}{2}\epsilon_0 E_0^2, T_{zz} = 0$
- 64. What is the contour integral of $f(z) = 1 + z^2$ in anti-clockwise sense over a unit square with corners at the four points (0,0)(1,0), (0,1) and (1,1) at the origin in the complex plane?
 - (A) $8\pi/3$
 - (B) $2\pi \left(1 + \frac{1+x}{3}\right)$
 - (C) $2\pi \left(1 + \frac{1-x}{3}\right)$
 - **(D)** 0
- 65. There are N non-interacting, identical but distinguishable particles in equilibrium at temperature T. Each of the particles can be in only two energy levels. One of them which is non-degenerate has energy zero (0), while the other is doubly degenerate with energy $2k_BT$. Find the approximate value of N if the mean energy of the system is $100 k_BT$
 - (A) 34
 - **(B)** 235
 - **(C)** 1000
 - **(D)** 540

- 66. A half-wave plate (HWP) is introduced between two crossed polaroids P1 and P2. The optics axis of (HWP) makes an angle 15° with the axis of P1. If an unpolarized beam of intensity I_{\circ} is normally incident on P1 and I_2 is the intensity after P2 then I_2/I_{\circ} is given by
 - (A) $\frac{l_2}{I_0} = \frac{1}{8}$
 - (B) $\frac{I_2}{I_0} = \frac{1}{4}$
 - (C) $\frac{l_2}{l_0} = \frac{1}{2}$
 - (D) $\frac{I_2}{I_0} = \frac{1}{6}$
- 67. In a FET V_{P1} is the pinch-off voltage for a gate-source voltage $(V_{GS}) =$. If the gate is now reverse biased, then which of the following is true for the pinch-off voltage V_{P2} at this value of $_{GS}$
 - (A) $V_{P2} < V_{P1}$
 - (B) $V_{P2} > V_{P1}$
 - (C) $V_{P2} = V_{P1}$
 - (D) V_{P2} is independent of V_{P1}
- 68. In an abrupt p n junction, the doping concentrations on the *p*-side and *n*-side are $N_A = 9 \times {}^{16}/\text{cc}$ and $N_D = 1 \times 10^{16}/\text{cc}$. If the p n junction is increase biased and the total depletion width is 3 μ m, then the value of depletion width on the donor side is
 - (A) 2.7 μm
 - **(B)** 0.3 μm
 - **(C)** 3 μm
 - (D) $0.27 \ \mu m$

69. The dual of the Boolean function $F = A\overline{B} + AC + BC$ is

- (A) $(\bar{A} + B)(\bar{A} + \bar{C})(\bar{B} + \bar{C})$
- (B) $(A + \bar{B})(A + C)(B + C)$
- (C) $\bar{A}B + \bar{A}\bar{C} + \bar{B}\bar{C}$
- (D) $\overline{A}\overline{B} + \overline{A}\overline{C} + B\overline{C}$
- 70. The state of a J K flip flop goes from "1" to "0" after applying a clock pulse (that is Q(t) = 1 and Q(t + 1) = 0). Which of the following is the correct and complete input combination?
 - (A) J = 0 and K = 1
 - (B) J = 0 and K = X
 - (C) J = X and K = 1
 - (D) J = 1 and K = 1

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71. A transmission line of length of l has characteristic impedance A_o . The line is cut into half. The value of characteristic impedance becomes

1.1.**3** - N.

- (A) $\frac{Z_o}{2}$
- (B) $\frac{Z_o}{4}$
- (C) iZ_o
- (D) Z_o
- 72. Antenna 1 has radiation resistance twice that of antenna 2. It implies that
 - (A) Antenna 2 delivers double power to space from antenna 1.
 - (B) Antenna 2 delivers half power to space than antenna 1.
 - (C) Antenna 2 delivers quarter power to space than antenna 1.
 - (D) Antenna 2 delivers equal power to space as antenna 1.
- 73. For a two-dimensional free electron gas, the electron density n and the Fermi energy E_F are related by

(A)
$$n = \frac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$$

(B) $n = \frac{mE_F}{\pi\hbar^2}$

(C)
$$n=rac{mE_F}{2\pi\hbar^2}$$

(D)
$$n = \frac{(2mE_F)^{1/3}}{\pi\hbar}$$

- 74. The temperature (T) dependence of magnetic susceptibility (χ) of a ferromagnetic substance with a (Curie) temperature (T_c) is given by
 - (A) $\frac{C}{T-T_c}$, for $T < T_c$
 - (B) $\frac{C}{T-T_c}$, for $T > T_c$
 - (C) $\frac{C}{T+T_c}$, for $T > T_c$
 - (D) $\frac{C}{T+T_c}$ for all temperatures, where C is a constant.
- 75. A flat circular disc of radius *a* has uniform charge density $-\sigma$ from its center upto a certain radius *r* and uniform surface charge density $+\sigma$ from *r* to *a*. Total charge on disc is zero. If the disc is moving with angular velocity ω about its axis passing through the center, find its magnetic moment *m*
 - (A) $\frac{\sigma\pi\omega a^4}{2}$
 - (B) $\frac{\sigma\pi\omega a^4}{4}$
 - (C) $\frac{\sigma\pi\omega a^4}{8}$
 - (D) $\frac{\sigma\pi\omega a^4}{16}$