ENTRANCE EXAMINATION – 2016

M.Sc. Statistics

Hall Ticket Number

Time : 2 hours
Max. Marks. 100

Part A : 25 marks
Part B : 75 marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR answer sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet after the examination.
5. There are plain sheets in the booklet for rough work, no additional sheets will be provided.
6. Calculators are not allowed.
7. There are a total of 50 questions in Part A and Part B together.
8. **Each question in Part – A has only one correct option and there is negative marking of 0.33.**
9. **There is no negative marking in Part - B. Some questions have more than one correct option. All the correct options have to be marked in the OMR answer sheet, otherwise zero marks will be credited.**
10. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
11. **THE MAXIMUM MARKS FOR THIS EXAMINATION IS 100 AND THERE WILL BE NO INTERVIEW.**
12. Given below are the meanings of some symbols that may have appeared in the question paper: $\mathbb{R}$-The set of all real numbers,$E(X)$-Expected value of the random variable X, $V(X)$-Variance o the random variable X, $Cov.(X,Y)$-Covariance of the random variables X and Y, iid-independent and identically distributed, pdf-probability density function, $B(n,p)$ and $N(\mu,\sigma^2)$ denote respectively,the Binomial and the Normal distributions with the said parameters.$Rank(A)$ and $det(B)$ mean rank and determinant of the matrices A and B respectively.

K- 7

Part - A

- **Find the correct answer and mark it on the OMR sheet. Each correct answer gets 1 (one) mark and wrong answer gets -0.33 marks.**

1. A and B are two independent events whose probabilities are strictly greater than zero and strictly less than one. Identify the wrong statement.
   (a) A and B^c are independent. (b) A^c and B^c are independent.
   (c) $P(A|B) = P(A|B^c)$. (d) For any event C with $0 < P(C) < 1$, $P(A \cap B|C) = P(A|C)P(B|C)$.

2. The number of different arrangements of 10 balls of which $m > 0$ are red and the rest are blue is maximum for m equal to
   (a) 6. (b) 5. (c) 4. (d) 3.

3. The marks of a class of 100 students are positively skewed, the average marks are 62%, so,
   (a) more than half of the students in this class have got less than 62% marks.
   (b) half of the students in this class have got more than 62% marks.
   (c) more than 62 students have got more than 70% marks.
   (d) exactly 50 students have got more than 62% marks.

4. The mean and median incomes of 100 households in a locality were both equal to $Rs.50,000$, 6 households whose incomes were $Rs.28000, 55000, 30000, 52000, 26000, 54000$ left the locality and in their places 6 new households joined the locality and their incomes are $Rs30000, 40000, 28000, 60000, 62000, 72000$, with these changes
   (a) there is no change in either the mean income or the median income.
   (b) the mean and median incomes of the locality have increased but both are still equal.
   (c) the median income is unchanged, but the mean income of the locality has increased.
   (d) the median income has reduced, but the mean income of the locality has increased.

5. Two coins with probabilities of heads p_1 and p_2 are tossed simultaneously, and if Pr(both coins show up tails) $= Pr$(both coins show up heads), then $p_1 + p_2$ is equal to
   (a) 1. (b) 3/4. (c) 1/2. (d) 1/4.

6. If $X \sim B(30, 0.5)$, then
   (a) $Pr(X > 15) < 0.5$. (b) $Pr(X > 15) > 0.5$. (c) $Pr(X > 15) = 0.5$. (d) $Pr(X < 15) = 0.5$.

K-7

7. Given in the table is the frequency distribution of a random variable say, X

X	2	4	6	8	Total
Frequency	m-1	m-1	2m+3	2m+1	50

, the Mode is

(a) not unique. (b) 19. (c) 2 or 4. (d) 6.

8. Which of the following describes a Poisson random variable correctly?

(a) The number of red balls in a collection of 100 balls drawn with replacement from a big bag containing 10,000 red and 20,000 blue balls.

(b) The number of heads that show up when a fair coin(probability of heads 1/2) is tossed 5 million times.

(c) The number of people with a super high IQ(Intelligence Quotient), say over 180 in a highly populous country.

(d) the number of rolls of a fair die till 2 consecutive 6s show up.

9. The total income of all the 100 households in a locality is $Rs.25,000,000$, the expected value of the total income of a simple random sample without replacement of 10 households is

(a) $Rs.250,000$ (b) $Rs.2500,000$ (c) $Rs.25,000$ (d) $Rs.2500$

10. Which of the following is not an observation of a Binomial random variable?

(a) The number of heads that show up in 20 tosses of a biased coin.

(b) The number of times at least one 6 shows up when two dice, one fair and one biased are rolled simultaneously 10 times.

(c) The number of times a red ball is drawn in 5 draws with replacement from a bag containing 50 red and 100 blue balls.

(d) The total number of heads that show up when a fair coin is tossed 10 times and then a biased coin is tossed 6 times.

11. Two fair coins are tossed simultaneously till heads shows up on at least one of the coins, the probability that these coins have to be tossed at least 4 times is

(a) 1/64. (b) 1/8. (c) 1/16. (d) 3/256.

12. $X \sim N(0,1)$, the correlation coefficient between X and $Y = |X|$ is

(a) -1. (b) 1. (c) 0. (d) $-1/2$.

K-7

13. The correlation coefficient between two random variables X and Y is 1, and the variance of Y is four times that of X, when X is 5, Y is 12, if X is observed to be 12, what can you say about Y?
(a) Can't say anything based on the data given. (b)Y is equal to 24.
(c) Y will be 24 or 26 with probabilities 1/2 each. (d) Y is equal to 26.

14. For any independent random variables X_1 and X_2 whose expected values are zero and variances are 25, identify the incorrect statement:
(a) $X_1 + X_2$ and $X_1 - X_2$ are always independent. (b) $V(X_1 - X_2) = 50$.
(c) $V(X_1 + X_2) = V(X_1 - X_2)$. (d) $Cov(X_1 + X_2, X_1 - X_2) = 0$.

15. $P(Z \leq 2) = 0.97725$, where Z is the standard normal random variable. If the marks in a public exam are normally distributed with mean 45 and standard deviation 10, the proportion of students who got more than 70% marks
(a) is equal to 0.02275. (b) is more than 0.025.
(c) is less than 0.02275. (d) can not be determined.

16. In order to estimate the average number of movies seen by individuals in city per month, it was decided to stratify the population into disjoint strata to obtain more reliable estimates, a good criterion to stratify all the people of the city is
(a) Age. (b) Height. (c) Weight. (d) Favorite Colour.

17. Based on a random sample of size n, denote the α - level critical region for a hypothesis testing problem H_0 vs. H_1 as C_α, if we change the level to $\alpha_1 < \alpha$, then
(a) $C_{\alpha_1} = C_\alpha$ (b) $C_{\alpha_1} \subseteq C_\alpha$ (c) $C_\alpha \subseteq C_{\alpha_1}$ (d) $C_{\alpha_1} \cap C_\alpha = \emptyset$.

18. $X_1 \sim U((0,1))$ and X_2 is another random variable whose probability density function (pdf) is $f_2(x) = \begin{cases} 12x^2(1-x) & 0 < x < 1 \\ 0 & o.w \end{cases}$, identify the correct statement about the probability distributions of X_1 and X_2.
(a)$E(X_1) < E(X_2)$. (b)$Pr(0 < X_1 \leq 1/2) < Pr(0 < X_2 \leq 1/2)$.
(c)$E(X_1) = E(X_2)$. (d)$Pr(0 < X_1 \leq 1/2) = Pr(0 < X_2 \leq 1/2)$.

19. Two heads showed up in n tosses of a fair coin, the maximum likelihood estimate for n is
(a) 5 or 6. (b) 3 or 4. (c) 7. (d) 8.

20. The value of $\lim_{n\to\infty} (1 - \frac{1}{\sqrt{n}})^n$
(a) is equal to 0. (b) is equal to $e^{-3/2}$.
(c) is equal to e^{-1}. (d) does not exist.

K-7

21. When we say that $(39, 69)$ is a 95% confidence interval for a parameter θ based on a sample $X_1, \ldots, X_{50}$ of size 50, it means that

(a) the probability that $\theta \in (39, 69)$ is 0.95.

(b) the random interval $(L(X_1, \ldots, X_{50}), U(X_1, \ldots, X_{50}))$ contains θ for 95% of all samples of size 50 and 39 and 69 are the observed values of $L(X_1, \ldots, X_{50})$ and $U(X_1, \ldots, X_{50})$ respectively for the given sample.

(c) there is a small probability of .05 that $\theta < 39$ or $\theta > 69$.

(d) the probability that $\theta < 39$ is at most 0.05.

22. Which of the following is the most suitable approximation for the number of heads in very large number of tosses of a fair coin:

(a) Poisson. (b) Negative Binomial. (c) Normal. (d) Exponential.

23. X_1, X_2, X_3 is a random sample from a random variable for which $Pr.(X = 1) = p$; $Pr.(X = 0) = 1 - p$, $0 < p < 1$, the statistic $T(X_1, X_2, X_3) = X_1 + X_2 - X_3$ is

(a) an unbiased and sufficient estimator for p. (b) an unbiased but not a sufficient estimator for p.

(c) the most efficient estimator for p. (d) a sufficient but not an unbiased estimator for p.

24. The effectiveness of any treatment for diabetes may be effected by the weight of the patient, so 4 particular treatments were given for 3 months each to 4 patients each randomly in three weight groups - Normal, Heavy, Very Heavy, the percentage decrease in fasting sugar levels was observed in all the patients after 3 months of their respective treatments. This Designed Experiment is a

(a) Completely Randomized Design (CRD) and 48 patients are required for this study.

(b) Randomized Block Design (RBD) meant to test whether each treatment is equally effective in each weight group.

(c) RBD for testing whether patients in the different weight groups respond similarly to each of the 4 treatments.

(d) RBD for testing whether each treatment is equally effective on an average across the weight groups.

25. "Ashok secured top grades in at least two of the five courses" is equivalent to

(a) Ashok did not get top grades in three courses.

(b) Ashok did not get top grades in at least three courses.

(c) Ashok secured top grades in all the five courses.

(d) Ashok did not get top grades in at most three courses.

K-7

Part - B

- **Questions (26)-(37) have more than one correct option. For the answer to be right all the correct options have to be marked on the OMR sheet. No credit will be given for partially correct answers.**
- **Questions (38)-(50) have only one correct option.**
- **Find the correct answers and mark them on the OMR sheet. Correct answers (marked in OMR sheet) to a question get 3 marks and zero otherwise.**

26. The sum of 7 positive real numbers $a_1, \dots, a_7$ is 14, so

    (a) their product $a_1a_2a_3a_4a_5a_6a_7$ can be 125.

    (b) their product $a_1a_2a_3a_4a_5a_6a_7$ can not be be 130.

    (c) if 5 of $a_1, \dots, a_7$ are greater than 2, the sum and the product of the remaining two are both less than 4.

    (d) unless all of $a_1, \dots, a_7$ are equal, at least one of them is greater than 2.

27. An odd number is twice as likely to be selected as an even number from $1, \dots, 10$, one number is selected, let A, B and, C be the events that an even number is selected, number selected is a multiple of 3 and that the selected number is a multiple of 5 respectively. Identify the correct statements:

    (a) A and B are independent events. (b) The expected value of the selected number is more than 5.

    (c) A and C are independent events. (d) $P(B|A) < P(B)$.

28. For any pair of *iid* random variables X_1 and X_2 that are symmetric about zero, identify the pairs that are identically distributed

    (a)$X_1 + X_2$ and $X_1 - X_2$. (b) $X_1 - X_2$ and $X_2 - X_1$.

    (c)$2X_1 + X_2$ and $X_1 + 2X_2$. (d) X_1 and $\frac{X_1+X_2}{2}$.

29. In a family that has three children, it is known that there is at least one girl child, a child is as likely to be a girl as a boy so, the probability that

    (a) the other two are boys is more than the probability that the other two are girls.

    (b) the other two are boys is equal to the probability that the other two are girls.

    (c) there are two girls is equal to the probability that there are boys in this family.

    (d) there are more girls is more than the probability that there are more boys in this family.

K-7

30. The Lowest, Average and Highest marks obtained by students in two classes A and B are given below:

Class	Lowest	Average	Highest
A	40	55	95
B	35	50	85

If we send the student who got 95 marks from $Class - A$ to $Class - B$ and one who got 85 marks from $Class - B$ to $Class - A$, we have

(a) reduced the difference in the average marks of the two classes.

(b) increased the variation in marks in at least one of the classes.

(c) reduced the variation in marks in both the classes.

(d) reduced the variance in the marks of all the students.

31. The heights of Indian adult males are normally distributed with mean $167cm.$ and standard deviation $7cm.$, so

(a) half of the Indian adult males are taller than $167cm.$.

(b) about as many are taller than $184cm.$ as those shorter than $153cm.$.

(c) if there are 100000 Indian adult males, there may be at most 14 of them who are taller than $188cm,$.

(d) half of them are shorter than $160cm.$.

32. H_0 and H_1 are the null and alternate hypotheses regarding a random variable, some observations are made of this random variable, the probability of getting the observations that we have if H_0 is true is 0.98 and if H_1 is true, the probability of getting this data is 0.4,

(a) this implies that H_0 is true.

(b) this implies that we should accept H_0 because H_1 is false.

(c) we should accept H_0 because data contain significantly more evidence in favour of H_0 than for H_1.

(d) the p- value for this test will be large.

33. 45 people are employed in a company, the highest income is drawn by 5 of them while 10 each of them draw the second, third, fourth highest and the remaining 10 draw the least income, to estimate the minimum and maximum incomes of this company, it was decided to draw a simple random sample without replacement of 10 employees and use the minimum income and maximum income in the sample as estimators respectively. Identify the correct statements.

   (a) the probability of there being an individual with the highest income in the sample is a little less than 1/2.

   (b) the sample minimum and maximum incomes are not unbiased estimators of the minimum and maximum wages respectively.

   (c) the sample minimum overestimates the minimum income, while the sample maximum underestimates the maximum income.

   (d) if the highest, second highest, third highest and fourth highest incomes are 5, 4, 3 and 2 times as much as the minimum income, the expected value of the sample maximum is almost half of the maximum income.

34. $X_1, \ldots, X_n$ is a random sample from the random variable whose pdf is $f(x) = \begin{cases} \lambda e^{-\lambda(x-\mu)} & \mu < x < \infty \\ 0 & o.w \end{cases}$,
let $\bar{X} = \frac{1}{n}(X_1 + \ldots + X_n)$, $X_{(1)} = min.\{X_1, \ldots, X_n\}$, $X_{(n)} = max.\{X_1, \ldots, X_n\}$, then,

   (a) If $n = 2$, $E(X_{(2)} - X_{(1)}) = \frac{1}{\lambda}$.

   (b) $X_{(1)}$ is a sufficient estimator for μ whether λ is known or not.

   (c) $\frac{n}{n-1}(\bar{X} - X_{(1)})$ is an unbiased estimator for $\frac{1}{\lambda}$.

   (d) $(n\bar{X}, X_{(1)})$ is jointly sufficient for (λ, μ).

35. f_1 and f_2 pdfs of two random variables X_1 and X_2 which are defined as $f_1(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty$ and $f_2(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty$, identify the correct statements

   (a) $Pr(|X_1| > 2) < Pr(|X_2| > 2)$.

   (b) $E(X_1^{2n-1}) = E(X_2^{2n-1}) = 0, \quad \forall n = 1, 2, \ldots$.

   (c) $E(X_1^{2n}) = E(X_2^{2n}), \quad \forall n = 1, 2, \ldots$.

   (d) The moment generating functions of X_1 and X_2 exist and are equal.

K-7

36. The function F defined on $\mathbb{R}$ as $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} + \frac{x}{2} & 0 \leq x < 1 \\ 1 & x > 1 \end{cases}$, then F is

(a) bounded.

(b) non-decreasing in x.

(c) continuous everywhere.

(d) not continuous at $x = 0$, but continuous everywhere else.

37. Consider a 3×3 real matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ a & 2 & 2 \end{pmatrix}$, then

(a) $Rank(A) = 3$, whatever a maybe.

(b) $det(A) \neq 0$ for some choices of a.

(c) $Rank(A) = 2$, whatever a maybe.

(d) there is no $a \in \mathbb{R}$ for which $Rank(A) = 1$.

38. There are 5 balls in a bag and it is equally likely that $1, 2, 3, 4$, or all 5 of them are red, if two balls are drawn without replacement from this bag, the probability that at least one of them is red is (a)0.5. (b)0.6. (c)0.72. (d)0.8.

39. All 100 students in a class in UoH know at least one of the three languages, Telugu, Hindi and English. 80 know Telugu, 40 know Hindi and 60 know English, further, 20 know both Telugu and Hindi, 50 know both Telugu and English, while 25 know both Hindi and English, then the number of students who know Telugu and English but not Hindi is

(a) 35. (b) 25. (c) 20. (d) 18.

40. Two randomly selected squares are to be removed from a chess board, if every pair to be removed is equally likely, the probability that the two removed squares are neither in the same row, nor in the same column, nor in the same diagonal-main or off is

(a) less than 1/3. (b) more than 1/3, but, less than 2/3.

(c) more than 2/3, but, less than 3/4. (d) more than 3/4.

K-7

41. The probability distribution of a random variable X is $Pr(X=j)=\frac{1}{j(j+1)}$, $j=1,2,\ldots$, regarding its moments,

(a) the first raw moment exists, none of the raw moments of order higher than 1 exist.

(b) the first and second raw moments exist.

(c) all the raw moments exist.

(d) none of the raw moments exist.

42. Select two numbers without replacement from $1,\ldots,10$, assume that every selection of two numbers is equally likely, what is the probability that the product of the selected numbers is at least 20 given that at least one of the selected numbers is a prime number?

(a) 15/30. (b) 16/30. (c) 17/30. (d) 18/30.

43. The expected value of a random variable X is 55, and the probability that this random variable takes values between 25 and 85 is 0.98, so

(a) the variance of X could be 20.

(b) the variance of X is 18.

(c) the variance of X is less than 15.

(d) nothing can be said about the variance of X based on the information given.

44. The mean of a Poisson random variable X is 1/2, then, $E(\frac{1}{X+1})$ is
(a) 2 . (b) 3. (c) less than 1. (d) more than 3.

45. X_1 and X_2 is a random sample from the Geometric distribution, with $Pr(X_i=j)=p(1-p)^{j-1}$, $i=1,2$; $j=1,2,\ldots$ $0<p<1$, an unbiased estimator for $\frac{p}{1+p}$ is
(a) $2^{-(X_1+X_2)}$. (b) $\frac{1}{2}(\frac{1}{2^{X_1}}+\frac{1}{2^{X_2}})$. (c) $2^{-(X_1+X_2-1)}$. (d) $\frac{1}{2^{X_1}}+\frac{1}{2^{X_2}}$.

46. X_1 and X_2 are independent exponential random variables, with means 1/2 and 1/3 respectively, then $Pr(X_1<X_2)$ is
(a) 1/5. (b) 2/5. (c) 4/5. (d) 1.

K-7

47. Let X be a random sample of size one from the $U(\theta, \theta+1)$ distribution, $\theta \in \mathbb{R}$. For testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$, the test is to reject H_0 if $X > 1.5$, its

(a) Power $= 1/2$ and Size $= 1$.

(b) Power $= 1/2$ and Size $= 1/2$.

(c) Power $= 1$ and Size $= 1/2$.

(d) Power $= 1$ and Size $= 0$.

48. Let X_1, X_2, X_3 be a random sample from the $N(0, \theta^2)$ population, $\theta > 0$. Then, the value of k, for which the estimator $k(|X_1| + |X_2| + |X_3|)$ is an unbiased for θ is

(a) $\frac{1}{3\pi}$. (b) $\sqrt{\frac{2}{9\pi}}$. (c) $\sqrt{\frac{\pi}{18}}$. (d) $\frac{2}{3\pi}$.

49. $2.8, 2.1, 2.6, 2.4, 1.9, 2.7$ is a random sample from the $U(\theta, \theta+1)$, $\theta > 0$ distribution, the maximum likelihood estimate for θ based on this sample is

(a) unique and is equal to 1.9.

(b) unique and is equal to 2.4.

(c) 1.9 or 2.

(d) every number in the interval $[1.8, 1.9]$.

50. A is a 5×5 real matrix, and its rank is 5, B is another 5×5 real matrix in which the first and fifth rows are the fifth and first rows of A respectively, the second and the fourth rows are the fourth and second rows of A respectively, while third row is the third row of A, so certainly,

(a) $Rank(B) = 5, Rank(A+B) = 3$.

(b) $Rank(B) = 5, Rank(A+B) = 5$.

(c) $Rank(B) = 4, Rank(A+B) = 4$.

(d) $Rank(B) = 4, Rank(A+B) = 5$.

K-7