Signature of Invigilator: __________

University of Hyderabad
Entrance Examination, February 2013
Question Paper
Ph.D. (ACRHEM)

Marks: 75
Time: 2.00 Hrs.

Hall Ticket No. ________________

Please confirm that

(a) This booklet has all 18 pages (including 2 blank pages) printed clearly and numbered

(b) You are given a clean and clear OMR sheet.

Read **carefully all** the instructions given below & on the OMR sheet

1. Please enter your Hall ticket number on Page 1 **(this sheet)** of this booklet **without fail.**
2. Please enter your Hall ticket number on the **OMR sheet without fail.**
3. All answers are to be marked on the OMR answer sheet following the instructions provided on the OMR sheet.
4. **No additional sheets will be provided.** Rough work is to be done in the booklet itself/ space provided on pages **17, 18**
5. **Handover** the fully answered **OMR answer sheet** at the end of the examination **without fail.**
6. Question paper has two parts: **Part-A** and **Part-B.**
7. **Part-A** consists of 25 objective type questions of one mark each. **There is negative marking of 0.33 marks for every wrong answer.** The marks obtained by the candidate in this part will be used for resolving tie cases.
8. **Part-B** consists of three sections **Physics (26-50), Chemistry (51-75) & Mathematics (76-100)** each containing 25 questions.
9. One needs to **answer any 25 questions from Part-B**. Each correct answer carries two marks. There is no negative marking in the sections of Part-B.
10. **In case the number (N) of answered questions is greater than 25, in Part-B, then the marks per question shall be 50/N.**
11. Non programmable calculators are permitted.
12. All the symbols have their usual meanings.

PART-A

1. A very slow moving 1 ton car (KE ~ 0) encounters a sinusoidal bump in the road which is 1ft high and 100ft long. The probability that the car can overcome the bump is

   A) exp(-14.3×10^{31}) B) exp(-1.9×10^{18}) C) exp(-16.2×10^{36}) D) exp(-6.2×10^{23})

2. The time – bandwidth product for a Gaussian pulse with a linear chirp of $2\sqrt{2}$ is

   A) 2ln6/π B) ln2/π C) ln6/π D) 6ln2/π

3. Two parallel wired carry currents I_1 and $2I_1$ in opposite directions. The magnetic field at a point midway the wires is

   A) $\frac{3I_1\mu_0}{2\pi r}$ B) $\frac{3I_1\mu_0}{\pi r}$ C) $\frac{I_1\mu_0}{3\pi r}$ D) $\frac{I_1\mu_0}{2\pi r}$

4. In Rutherford's scattering experiment the maximum kinetic energy of alpha particles was 7.7 MeV. Assume that such an alpha particle approaches the target nucleus with b=0. How close to the target nucleus can this alpha particle go?

   A) 3×10^{-14} m B) 1.5×10^{-14} m C) 0.33×10^{-14} m D) 6×10^{-14} m

5. The optical property of DNA is made use of in

   A) Optical waveguiding B) Quartz crystal manufacture
   C) Optical switch fabrication D) Light emitting diodes

6. The family of ferromagnets with sizable magnetic moment are

   A) Co, Ni, Mn B) Cu, Ni, Fe C) Co, Ni, Fe D) Co, Cu, Fe

7. When the mirror in Michelson interferometer is displaced, the interference pattern is shifted by 100 bands. Given light of λ = 600 nm used in the experiment, what is the mirror's displacement?

   A) 20 μm B) 35 μm C) 25 μm D) 30 μm

8. Emission bands in metals are

   A) Perfectly sharp and emission edge remains constant with temperature
   B) Not perfectly sharp and emission edge remains constant with temperature
   C) Not perfectly sharp and emission edge varies with temperature
   D) Perfectly sharp and emission edge varies with temperature

9. The unit cell given by following parameters a=1.06 Å, b=0.948 Å, c=0.65 Å, α=44°, β=88° and γ=96° describes the following crystal system

   A) Monoclinic B) Triclinic C) Hexagonal D) Tetragonal

10. A certain amount of water of heat capacity C is at temperature of 0 °C. It is placed in contact with a heat reservoir at 100 °C and the two come to thermal equilibrium. The change in the entropy of the universe is

A) $0.088C$ B) $0.044C$ C) $0.022C$ D) $0.066C$

11. The colored oxide of nitrogen among the following is

A) N_2O B) NO_2 C) NO (solid) D) N_2O_4

12. Stable metallocene among the following is

A) Cobaltocene B) Nickelocene C) Vanadocene D) Manganocene

13. Ground state term for Fe^{2+} ion in weak ligand field is

A) ^3F B) ^4F C) ^5D D) ^6S

14. The correct statement about Superacids is

A) They are aqueous solution of strong acids
B) They are non-aqueous in nature
C) Their pH is equal to zero
D) Their pH is greater than zero

15. Solutions of the following compounds, all at the same molality, were prepared. Which solution has the lowest freezing point?

A) KBr B) $Al(NO_3)_3$ C) $NaNO_2$ D) $MgCl_2$

16. Ground electronic term of N^{2+} is

A) $^2\Sigma$ B) $^3\Sigma$ C) $^1\Sigma$ D) $^2\Pi$

17. Eigen functions of L^2 operator are defined by

A) Laguerre Polynomials B) Hermite Polynomials
C) Associated Legendre Polynomials D) Jacobi Polynomials

18. Existence of molecular orbital can be confirmed by

A) IR Spectroscopy B) Microwave Spectroscopy
C) NMR Spectroscopy D) Photoelectron Spectroscopy

19. The value of $\oint \vec{A}.\overrightarrow{dr}$ around the circle $x^2 + y^2 = R^2$ in xy plane, given $\vec{A} = 2yi - 3xj + zk$ is

A) $-2\pi R^2$ B) $-4\pi R^2$ C) $-3\pi R^2$ D) $-\pi R^2$

20. The rank of the matrix $M = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is

A) 3 B) 4 C) 2 D) 1

21. Let $f : [a, b] \to \mathbb{R}$ be a monotonic function. Then

A) f is continuous
B) f is discontinuous at almost two points
C) f is discontinuous at finitely many points
D) f is discontinuous at most countable points

22. For a continuous piece wise smooth functions $u(x) and\ v(x)$ defined on [a,b] the following is true

A) $\int_a^b u'(x)v(x)\,dx = \int_a^b u(x)v'(x)\,dx - u(x)v(x)|_a^b$
B) $u(x)v(x)|_a^b = \int_a^b u'(x)v(x)\,dx - \int_a^b u(x)v'(x)\,dx$
C) $\int_a^b u'(x)v(x)\,dx = u(x)v(x)|_a^b + \int_a^b u(x)v'(x)\,dx$
D) $\int_a^b u'(x)v(x)\,dx = u(x)v(x)|_a^b - \int_a^b u(x)v'(x)\,dx$

23. The general solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is of the form
A) $u = f(x + y) + g(x - y)$
B) $u = cf(x - iy)$
C) $u = g(x + iy)$
D) $u = f(x + iy) + g(x - iy)$

24. The sequence defined as $g(n) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}$ is

A) Monotonically decreasing and lim g $\geq$ 2
B) Monotonically increasing, bounded and lim g $\leq$ 1
C) Monotonically decreasing and lim g $\geq$ 1
D) Monotonically increasing, bounded and lim g $\leq$ 2

25. The regular singular points for $z^2(z + 2)x'' + zx' - (2z - 1)x = 0$ are

A) 0, -2 B) 0, 2 C) -1/2, 2 D) 1, -1/2

PART – B

PHYSICS

26. An electron is accelerated from rest through a potential difference of 565 V. This electron enters a region, at an angle of 30° below the horizontal, where there is a uniform electric field of 35 V/cm directed downward. What is the direction of the travel of the electron at 5×10^{-8} s?

A) 62°45′ B) 27°55′ C) 152°45′ D) 117°55′

27. A 50 fs pulse centred at 248 nm is focused using a lens of 30 mm focal length (n=1.51, $\lambda dn/d\lambda$ = -0.17). The Chromatic aberration induced change in the focal length and the group delay picked up by the pulse are respectively

A) 45 μm, 600 fs B) 60 μm, 300 fs C) 75 μm, 225 fs D) 30 μm, 250 fs

28. Jones and Stokes vectors for a vertical polarized state are

A) $\begin{bmatrix}0\\1\end{bmatrix};\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$ B) $\begin{bmatrix}1\\0\end{bmatrix};\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$ C) $\begin{bmatrix}1\\1\end{bmatrix};\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$ D) $\begin{bmatrix}1\\i\end{bmatrix};\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$

29. The enclosed charge and electric flux through the surface of a sphere containing 15 protons and 10 electrons is

A) 8×10^{-19}C; 9.04×10^{-8} Vm
B) 8×10^{-19}C; 2×10^{-7} Vm
C) 2.4×10^{-19}C; 1.9×10^{-8} Vm
D) 8×10^{-18}C; 8.8×10^{-7} Vm

30. Phase delay induced by a lens of radius of curvature R, refractive index n, at a position (x,y) on the lens surface is

A) $(n-1)(x^2+y^2)/2R$
B) $-(n-1)(k/2R)(x^2+y^2)$
C) $-(nk/R)\sqrt{(x^2+y^2)}$
D) $R - (x^2+y^2)/2R$

31. What density of current would produce a magnetic field given by
$B = \left(\frac{a}{r}+\frac{b}{r}e^{-r}+ce^{-r}\right)\hat{\varphi}$ in cylindrical coordinates

A) $e^{-r}\left(\frac{a}{r}+\frac{b}{r}+c\right)\hat{z}$
B) $e^{-r}\left(-\frac{b}{r}+\frac{a}{r}-c\right)\hat{z}$
C) $e^{-r}\left(-\frac{b}{r}+\frac{c}{r}-c\right)\hat{z}$
D) $e^{-r}\left(-\frac{b}{r}+\frac{c}{r}-a\right)\hat{z}$

32. The reflection coefficient for scattering of a potential $V(x) = V_0 a\delta(x)$ is

A) $\frac{(maV_0)^2}{(\hbar^4k^2+m^2a^2V_0^2)}$ B) $\frac{maV_0}{(\hbar^2k^4+m^2a^2V_0^2)}$ C) $\frac{maV_0{}^2}{(\hbar^2k^4+m^2a^2V_0^2)}$ D) $\frac{(maV_0)^2}{(\hbar^2k^2+m^2a^2V_0^2)}$

33. The generating function and the corresponding transformation defined by the equations $P = 2\left(1 + q^{1/2}cosp\right)q^{1/2}sinp;\ Q = \ln\left(1 + q^{1/2}cosp\right)$ are

A) $(e^{P} - 1)^2 \tan p + C$, Lagrangian
B) $-(e^{Q} - 1)^2 \tan p + C$, Canonical
C) $-(e^{iQ} - 1)^{1/2} \tan p + C$, Eulerian
D) $-(e^{iP} - 1)^{1/2} \tan p + C$, Jacobian

34. A rectangular brass bar of mass M has dimensions a,b,c. The moment of inertia I is measured about an axis in the centre of the ab face and perpendicular to it. Considering the following measurements are made: $M = 135.0\pm0.1$ gm, $a = 80\pm1$ mm, $b = 10\pm1$ mm, $c = 20.00\pm0.01$mm; what is the percentage standard error in the density of the material and moment of inertia respectively?

A) 10%, 2.5% B) 7.5%, 2% C) 2.5%, 10% D) 10%, 7.5%

35. For a typical cavity of a He-Ne laser with $d = 20$ cm, $n_0 \approx 1$, $R_1 = 1$, $R_2 = 0.98$, $\alpha_1 \approx 1$, the FWHM of the spectrum and the frequency separation between adjacent longitudinal modes in MHz respectively are

A) 2.4, 750 B) 3.6, 900 C) 1.2, 750 D) 4.5, 600

36. A 2 eV electron encounters a barrier of height 5 eV. What is the probability that it will tunnel through the barrier of width 1 nm?

A) 8.1×10^{-9} B) 7.1×10^{-8} C) 8.1×10^{-8} D) 7.1×10^{-7}

37. The matrix $M = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ can be written in terms of the Pauli matrices as

A) $\left(\frac{\alpha+\delta}{2}\right)\sigma_x + \left(\frac{\beta+\gamma}{2}\right)\sigma_y + i\left(\frac{\beta-\gamma}{2}\right)\sigma_z + \left(\frac{\alpha-\delta}{2}\right)I$
B) $\left(\frac{\alpha-\delta}{2}\right)\sigma_x + \left(\frac{\beta-\gamma}{2}\right)\sigma_y + i\left(\frac{\beta+\gamma}{2}\right)\sigma_z + \left(\frac{\alpha+\delta}{2}\right)I$
C) $\left(\frac{\alpha-\delta}{2}\right)I + \left(\frac{\beta-\gamma}{2}\right)\sigma_x + i\left(\frac{\beta+\gamma}{2}\right)\sigma_y + \left(\frac{\alpha+\delta}{2}\right)\sigma_z$
D) $\left(\frac{\alpha+\delta}{2}\right)I + \left(\frac{\beta+\gamma}{2}\right)\sigma_x + i\left(\frac{\beta-\gamma}{2}\right)\sigma_y + \left(\frac{\alpha-\delta}{2}\right)\sigma_z$

38. Natural light falls on a system of two Polaroids the angle between whose optical axes is 45°. Neglecting losses due to reflection and considering 10% of light is lost due to all possible mechanisms in each Polaroid, by how much will the intensity of light be reduced?

A) 50% B) 80% C) 20% D) 25%

39. The wavelength of radiation emitted from adjacent vibrational energy levels of NO molecule assuming k = 1550 N/m and μ = 7.466 amu is

A) 4.34 μm B) 5.34 μm C) 6.34 μm D) 3.34 μm

40. The fraction of gas molecules having mean-free path in the range of λ and 2λ is

A) 0.46 B) 0.23 C) 0.69 D) 0.82

41. In a Hall experiment on Zinc, a potential of 4.5μV is developed across a foil of thickness 0.02 mm. When current of 1.5 A is passed in a direction perpendicular to magnetic field of 2.0T, the electron density per cubic meter is

A) 2.08×10^{-29} B) 3.14×10^{-30} C) 2.56×10^{-28} D) 1.04×10^{-27}

42. The density of copper is 8.94 gm/cc and atomic weight is 63.5/mole. The effective mass of electron is 1.01. The Fermi energy of Cu in eV assuming each atom gives one electron

A) 4.33 B) 2.17 C) 5.32 D) 6.97

43. The stopping power of a 4 MeV proton compared to a 8 MeV deuteron in same medium is

A) Larger B) Smaller C) Equal D) Zero

44. A blackbody has cubical shaped cavity. The number of modes of vibration per unit volume in the 499 – 501 nm region are

A) 8×10^{17} /m^3 B) 4×10^{17} /m^3 C) 1.6×10^{18} /m^3 D) 2×10^{17} /m^3

45. A crystal belongs to a face centred cubic lattice with four atoms in the unit cell. The structure factor for (0 1 0) and (2 0 0) reflection respectively are

A) 2f and zero B) Zero and 4f C) 2f and 2f D) 2f and 4f

46. Given three spin matrices

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{bmatrix}0 & 1 & 0\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}, S_y = \frac{\hbar}{\sqrt{2}}\begin{bmatrix}0 & -i & 0\\1 & 0 & -i\\0 & i & 0\end{bmatrix}, S_z = \hbar\begin{bmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & -1\end{bmatrix}$$

Commutator $[S_x, S_y]$ is

A) $-i\hbar S_z$ B) $-iS_z$ C) $i\hbar S_z$ D) S_z

47. Raman line associated with a vibrational mode which is both Raman and infrared active is found at 4600 A°. When sample is excited by light of wavelength 4358 A°, what will be the wavelength of the corresponding infrared band

A) 82838 A° B) 10640 A° C) 75320 A° D) 63286 A°

48. The normal modes of vibration of CO_2 molecules are 1330 cm^{-1}, 667 cm^{-1} and 2349 cm^{-1}. The zero point energy of CO_2 molecule is

A) 0.233 eV B) 0.311 eV C) 0.115 eV D) 0.460 eV

49. Given $I_E = 2.5$ mA, $h_{fe} = 140$, $h_{OC} = 20$ µs, the value of γ_e, h_{ie}, γ_0 in the common emitter hybrid equivalent circuit are

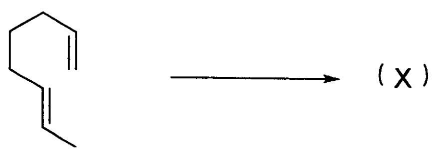
A) 10.4 Ω, 1.456 kΩ, 50 kA
B) 1.4 kΩ, 4.56 kΩ, 5 kA
C) 4.10 Ω, 50 kΩ, 1.5 kA
D) 10.4 kΩ, 1.456 Ω, 50 A

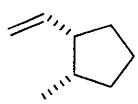
50. The pressure in pounds per square inch needed to isothermally compress an elastic solid to 20 times its normal density is

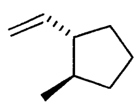
A) 43.45 B) 29.95 C) 34.54 D) 54.35

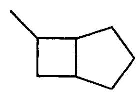
CHEMISTRY

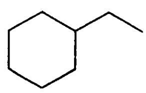
51. The major product (X) obtained in the following reaction is



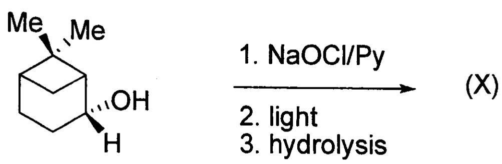
A) 

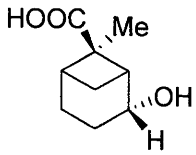
B) 

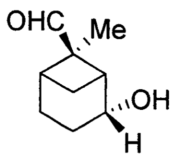
C) 

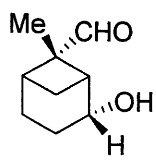
D) 

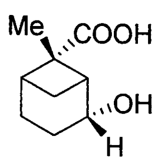
52. The major product (X) obtained in the following reaction is



A) 

B) 

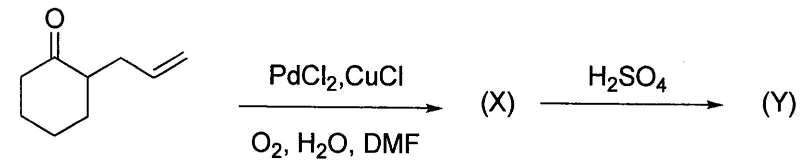
C) 

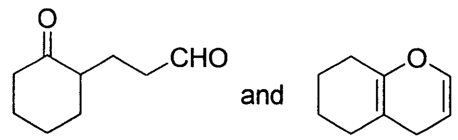
D) 

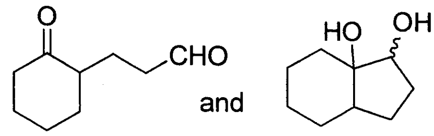
53. For which of the following sets of values of ΔH and ΔS will a reaction be spontaneous only at a high temperature?

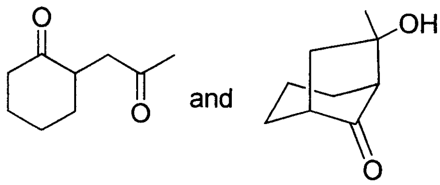
	ΔH (kJ)	ΔS (J/K)
(A)	+70	+30
(B)	+70	- 30
(C)	- 70	- 30
(D)	0	-30

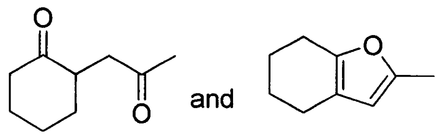
54. The major products (X) and (Y) obtained in the following reaction is



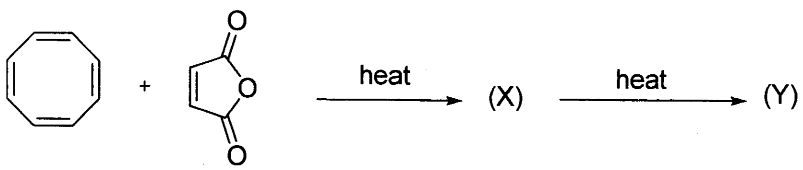
A) 

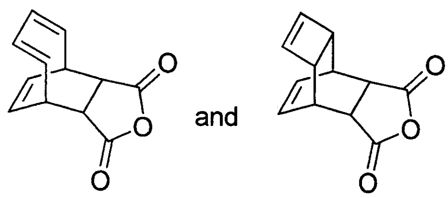
B) 

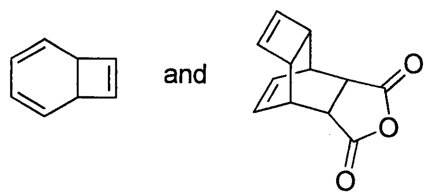
C) 

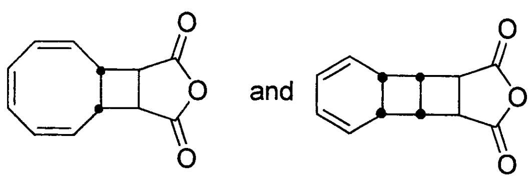
D) 

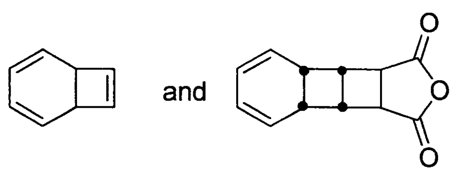
55. The major products (X) and (Y) obtained in the following reaction is



A) 

B) 

C) 

D) 

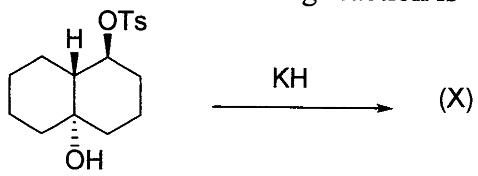
56. The ratio of spacing of 100, 110 and 111 planes of a body centered cubic lattice is

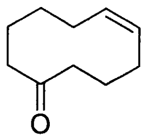
A) 1:0.707:0.577
B) 1:1.414:0.577
C) 1:0.707:1.514
D) 1:0.707:1.414

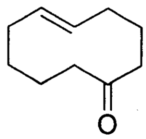
57. The gas pressure in an aerosol container is 1.5 atm at 25 °C. Assuming an ideal behavior of the gas, if the container is heated to 450 °C the pressure would be close to

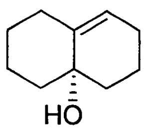
A) 1.023 atm
B) 1.234 atm
C) 3.639 atm
D) 2.639 atm

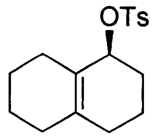
58. The major product (X) obtained in the following reaction is



A) 

B) 

C) 

D) 

59. $Hg^{2+} + 2e^- \rightarrow Hg$ $\quad E^0 = 0.85$ V
$Zn^{2+} + 2e^- \rightarrow Zn$ $\quad E^0 = -0.76$ V
Given the cell potentials shown above, the overall cell potential for the reaction $Zn + Hg^{2+} \Leftrightarrow Zn^{+2} + Hg$ is

A) 0.09 V
B) 1.61 V
C) 0.80 V
D) 0.18 V

60. Half life time ($t_{1/2}$) of a second order reaction is proportional to (where A_0 is the initial concentration of reactant)

A) A_0
B) A_0^2
C) $1/A_0$
D) $1/A_0^2$

61. The equivalent conductance ($ohm^{-1}cm^2eqv.^{-1}$) at infinite dilution (Λ_0) of acetic acid (HAc) will be [Given Λ_0 (HCl) = 420, Λ_0 (NaCl) = 126 and Λ_0 (NaAc) = 91]

A) 385
B) 637
C) 455
D) 203

62. The Miller indices of the face having intercept (1/2 a, 2 b, ∞ c; where a, b and c are the basis vector of the cubical lattice) is

A) (410) B) (220) C) (210) D) (012)

63. Gases A, B, C and D obey van der Waals gas equation with 'a' and 'b' (in suitable units) values as given in the table below

	A	B	C	D
a	6	6	20	0.05
b	0.025	0.15	0.10	0.02

Which of the gas has (i) the highest critical temperature, (ii) the largest molecular volume and (iii) the most ideal behavior at STP?

A) A, B and C B) A, B and D C) C, D and A D) A, D and B

64. An object of mass 200g is crushed into powder and spread over an area of 1.0 m^2 on Earth's surface. The pressure exerted by the powder is ($g = 9.81\ m\ s^{-2}$)

A) 200 Pa B) 1.96 Pa C) 1.69 Pa D) 9.62 Pa

65. Among the following the d-orbital that cannot be lowest in energy in a square planar complex is

A) d_{xz} B) d_{yz} C) d_{xy} D) d_{z^2}

66. The incorrect statement regarding Zigler Natta catalyst is

A) It contains Ti
B) It contains Al
C) It involves polymerization of terminal alkenes
D) It converts methane to methanol

67. Ligand to metal charge transfer occurs between

A) Metal in high oxidation state and ligand having low lying acceptor orbitals
B) Metal in low oxidation state and ligand having low lying acceptor orbitals
C) Metal in high oxidation state and ligand having lone pair of higher energy
D) Metal in low oxidation state and ligand having lone pair of moderate energy

68. Which of the following is incorrect regarding interpretation of spectra using Tanabe-Sugano diagram

A) It predicts spectra of coordination complexes more accurately than the Orgel diagram
B) It includes both low spin and high spin terms
C) The ground state is always taken as abscissa and the energy of the other states plotted relative to it
D) It does not include low spin terms

69. A 50.00 mL of 0.0050 M NaCl is titrated with 0.0100 M $AgNO_3$. Calculate pAg at 20 mL addition of $AgNO_3$ and at equivalence point. The reaction is $Ag+ + Cl- \leftrightarrows AgCl_{(s)}$ (for AgCl, $K_{sp}= 1.82 \times 10^{-10}$)

A) 4.87 and 6.79
B) 6.59 and 4.87
C) 0.05 and 0.1
D) 1.82 and 14

70. The total valence electrons in $(\mu\text{-CO})_2\text{-}[\eta^5\text{-CpRh}]_3CO$ is

A) 14 B) 16 C) 17 D)18

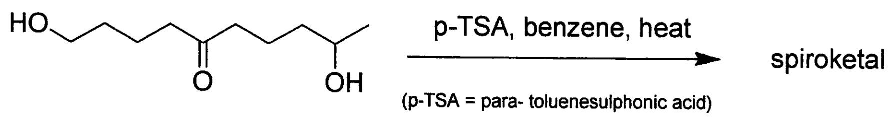
71. The V-C bond distances in $V(CO)_6^-$ and $V(CO)_6$ are 1.93 and 2.00 Å respectively. The correct justification in bond distances in these compounds is,

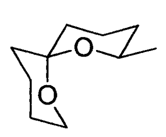
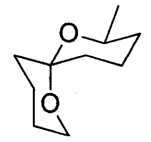
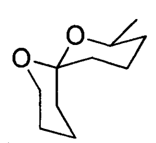
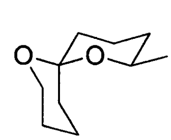
A) There is no back bonding in V(CO).
B) There is no back bonding in $V(CO)^-$.
C) The back bonding is less in $V(CO)^-$ than V(CO) because of the additional negative charge on V.
D) The back bonding is more in $V(CO)^-$ than V(CO) because of the additional negative charge on V.

72. Arrange the following ligands in the order of increasing σ donor capability.
PMe_3, $P(OEt)_3$, PPh_3, PF_3

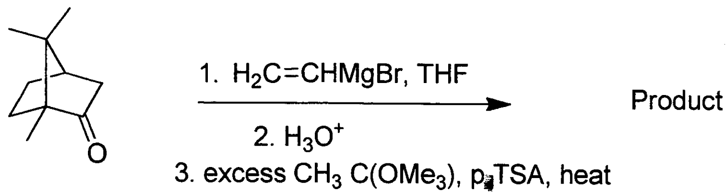
A) $PF_3 < P(OEt_3) < PPh_3 < PMe_3$
B) $PF_3 > P(OEt_3) > PPh_3 > PMe_3$
C) $PMe_3 < P(OEt)_3 < PPh_3 < PF_3$
D) $PMe_3 < P(OEt)_3 < PF_3 < PPh_3$

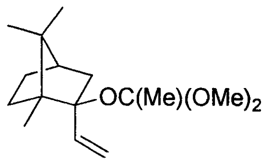
73. The most predominant spiroketal obtained in the acid catalyzed cyclization of 5-ketodecan-1,9-diol is

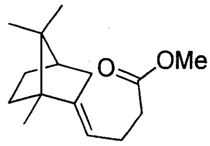


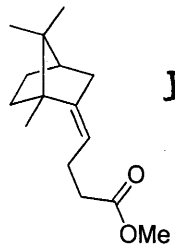
A) 
B) 
C) 
D) 

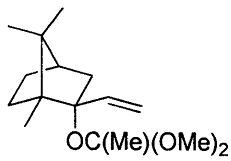
74. Identify the product in the following transformation



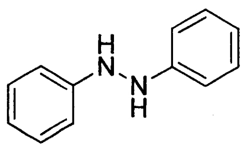
A) 

B) 

C) 

D) 

75. The major product obtain upon treating a mixture of the following compounds in a strongly acidic solution of H_2SO_4 is



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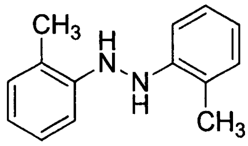
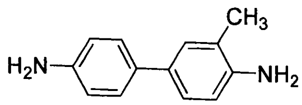


A) 

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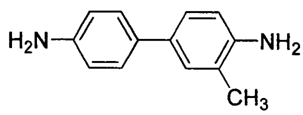
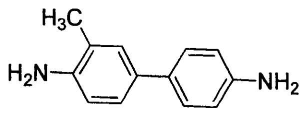


B) 

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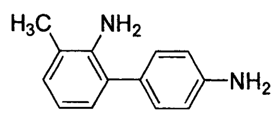
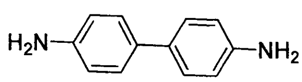


C) 

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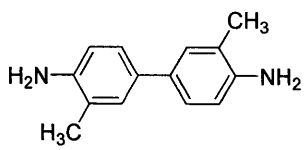
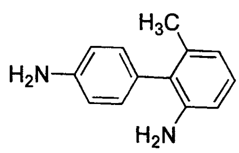


D) 

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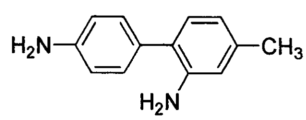

MATHEMATICS

76. A mass of 500 gm is attached to a spring whose spring constant is 200 Nm^{-1}. The mass is subjected to an initial displacement and velocity of 0.015 m and 0.4 ms^{-1} respectively. The equation of motion describing the spring-mass system is

A) $0.015\cos(20t) + 0.02\sin(20t)\, m$
B) $0.015\sin(20t) + 0.02\cos(20t)\, m$
C) $0.025\cos(20t + 0.93)\, m$
D) $0.015\cos(20t)\, m$

77. The Fourier series of the function $f(t) = \begin{cases} 0 & when -2 < t < -1 \\ k & when -1 < t < 1 \\ 0 & when\ 1 < t < 2 \end{cases}$, $T = 4$

A) $k + \frac{2k}{\pi}\left(\cos\frac{\pi}{2}t + \frac{1}{3}cos\frac{3\pi}{2}t - \frac{1}{5}cos\frac{5\pi}{2}t + \cdots\right)$
B) $k + \frac{2k}{\pi}\left(\cos\frac{\pi}{2}t - \frac{1}{3}cos\frac{3\pi}{2}t + \frac{1}{5}cos\frac{5\pi}{2}t - \cdots\right)$
C) $\frac{k}{2} + \frac{2k}{\pi}\left(\cos\frac{\pi}{2}t + \frac{1}{3}cos\frac{3\pi}{2}t - \frac{1}{5}cos\frac{5\pi}{2}t + \cdots\right)$
D) $\frac{k}{2} + \frac{2k}{\pi}\left(\cos\frac{\pi}{2}t - \frac{1}{3}cos\frac{3\pi}{2}t + \frac{1}{5}cos\frac{5\pi}{2}t - \cdots\right)$

78. $\int_{-1}^{1} P_2^1(x)P_2^1(x)\,(1-x^2)\,dx =$

A) 8
B) 6
C) 3
D) 1

79. For a Maxwellian distribution $\frac{dN}{N} = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} e^{(-mv^2/2kT)}V^2dV$, $\langle V^n\rangle$ is

A) $\frac{\left(\frac{2kT}{m}\right)^n\left(\frac{n+2}{3}\right)!}{\frac{1}{2}!}$
B) $\frac{\left(\frac{2kT}{m}\right)^{n/2}\left(\frac{n+1}{2}\right)!}{\frac{1}{2}!}$
C) $\frac{\left(\frac{2kT}{m}\right)^{n/3}(n+2)!}{3!}$
D) $\frac{\left(\frac{2kT}{m}\right)^{2n}\left(\frac{2n+3}{8}\right)!}{\frac{1}{2}!}$

80. The value of surface integral $I = \oiint (x^3dydz + x^2y\,dzdx + x^2z\,dxdy)$, where the closed surface consists of the cylinder $x^2 + y^2 = 9\ (0 \le z \le 4)$ and the circular disks $z = 0$ and $z = 4, (x^2 + y^2) \le 9$

A) 441π
B) 360π
C) 540π
D) 405π

81. The first half-range expansion of $f(t) = \begin{cases} \frac{2k}{l}t & when\ 0 < t < \frac{l}{2} \\ \frac{2k}{l}(l-t) & when \frac{l}{2} < t < l \end{cases}$

A) $f(t) = \frac{4k}{\pi^2}\left(\frac{1}{2}\cos\frac{2\pi}{l}t + \frac{1}{6}\cos\frac{6\pi}{l}t + \cdots\right)$
B) $f(t) = \frac{4k}{\pi^2}\left(\frac{1}{2}\cos\frac{\pi}{2l}t + \frac{1}{6}\cos\frac{\pi}{6l}t + \cdots\right)$
C) $f(t) = \frac{16k}{\pi^2}\left(\frac{1}{2^2}\cos\frac{2\pi}{l}t + \frac{1}{6^2}\cos\frac{6\pi}{l}t + \cdots\right)$
D) $f(t) = \frac{16k}{\pi^2}\left(\frac{1}{2}\cos\frac{\pi}{2l}t + \frac{1}{6}\cos\frac{\pi}{6l}t + \cdots\right)$

82. A Wronskian for $\emptyset_1 = e^x, \emptyset_2 = e^{-x}$ and $\emptyset_3 = \cosh x$ is

A) e^{2x}
B) 0
C) $e^x\cosh x$
D) $e^{-x}\cosh x$

83. The Laplace transform of $tsin\omega t$ is

A) $\frac{2\omega s}{(s^2+\omega^2)^2}$ B) $\frac{2\omega s}{(s^2-\omega^2)^2}$ C) $\frac{2\omega^2 s^2}{(s^2+\omega^2)^2}$ D) $\frac{2\omega^2 s^2}{(s^2-\omega^2)^2}$

84. For positive values of $x, \int \frac{dx}{x\sqrt{x^2-1}} =$

A) $-arc\cos\left(\frac{1}{x}\right) + C$ B) $-arc\cos(x) + C$

C) $-arc\sin(x) + C$ D) $-arc\sin\left(\frac{1}{x}\right) + C$

85. Value of $\oint\{(2xy^3 - y^2cosx)dx + (1 - 2ysinx + 3x^2y^2)dy\}$, where the closed loop is the arc of the parabola $2x = \pi y^2$ from (0,0) to (π/2,1) is

A) $\frac{\pi^2}{16}$ B) $\frac{\pi^2}{8}$ C) $\frac{\pi^2}{4}$ D) $\frac{\pi^2}{12}$

86. For the equality $(1 - Z)^{-1} = 1 + Z + z^2 + \cdots;\ Z = \varrho e^{i\theta}, \varrho < 1$ the Poisson and its conjugate kernel are

A) $P_\varrho(\theta) = \frac{1-\varrho^2}{1-\rho cos\theta+\varrho^2};\ Q_\varrho(\theta) = \frac{\varrho sin\theta}{1-\rho cos\theta+\varrho^2}$

B) $P_\varrho(\theta) = \frac{1}{2}\frac{1+\varrho^2}{1-\varrho^2};\ Q_\varrho(\theta) = \frac{1}{2}\frac{\varrho}{1+2\rho+\varrho^2}$

C) $P_\varrho(\theta) = \frac{1}{2}\frac{1-\varrho^2}{1+\varrho^2};\ Q_\varrho(\theta) = \frac{1}{2}\frac{\varrho}{1-2\rho+\varrho^2}$

D) $P_\varrho(\theta) = \frac{1}{2}\frac{1-\varrho^2}{1-2\rho cos\theta+\varrho^2};\ Q_\varrho(\theta) = \frac{\varrho sin\theta}{1-2\rho cos\theta+\varrho^2}$

87. The formula $A_0 f\left(\frac{-1}{2}\right) + A_1 f(0) + A_0 f\left(\frac{1}{2}\right)$ which approximates the integral $\int_{-1}^{1} f(x)\,dx$ is exact for polynomials of degree less than or equal to 2 if

A) $A_0 = A_1 = A_2 = 1$ B) $A_0 = A_2 = 4/3, A_1 = 2/3$

C) $A_0 = A_2 = 4/3, A_1 = -2/3$ D) $A_0 = A_2 = -4/3, A_1 = 2/3$

88. The orthogonal trajectories of the family of the parabolas $y = cx^2$ is given by

A) $\frac{x^2}{4} + \frac{y^2}{4} = c$ B) $\frac{x^2}{4} + \frac{y^2}{2} = c$ C) $\frac{x^2}{4} + \frac{y^2}{8} = c$ D) $\frac{x^2}{2} + \frac{y^2}{4} = c$

89. A random variable X has Poisson distribution. If 2P(X=2) = P(X=1) +2P(X=0), then the variance of X is

A) 3/2 B) 1/2 C) 1/3 D) 2/3

90. For the differential equation $4x^3y'' + 6x^2y' + y = 0$, the point at infinity is

A) a critical point B) an ordinary point

C) an irregular singular point D) a regular singular point

91. A metric space is always

A) First countable B) Second Countable C) Separable D) Connected

92. For a function $f(z) = \frac{z-\sin z}{z^3}$, the point $z = 0$ is

A) a pole of order 3
B) a pole of order 4
C) an essential singularity
D) a removable singularity

93. The harmonic conjugate of $u(x, y) = x^2 - y^2 + xy$ is

A) $x^2 - y^2 + xy$
B) $x^2 + y^2 - xy$
C) $\frac{1}{2}(y^2 - x^2) + 2xy$
D) $\frac{1}{2}xy + 2(y^2 - x^2)$

94. The Eigen values for the boundary value problem $y'' + \lambda y = 0, y(0) = 0,$ $y(\pi) + y'(\pi) = 0$ satisfy

A) $\sqrt{\lambda} + \tan\sqrt{\lambda}\pi = 0$
B) $\sqrt{\lambda} + \tan\lambda\pi = 0$
C) $\lambda + \tan\lambda\pi = 0$
D) $\lambda + \tan\sqrt{\lambda}\pi = 0$

95. Let $f(x, y) = \sqrt{|xy|}$, then

A) f_x and f_y do not exist at (0,0)
B) $f_y(0,0) = 0$
C) $f_x(0,0) = 1$
D) f is differentiable at the origin

96. Let U be an m-dimensional subspace of an n-dimensional vector space V, where m<n, then the dim (V/U) is

A) mn B) n-m C) m-n D) m/n

97. There exist zero divisors in

A) The ring of integers modulo a prime p
B) The ring of real matrices of order p
C) The ring of entire functions
D) The ring of polynomials

98. The singularity of $e^{\sin Z}$ at $Z = \infty$ is

A) a pole
B) a removable singularity
C) Isolated essential singularity
D) Nonisolated essential singularity

99. The general solution of $y'' - 4y' + 4y = 0$, given $y(0) = 3, y'(0) = 1$ is

A) $y = (3 - 5x)e^{2x}$
B) $y = (3x - 5)e^{2x}$
C) $y = (5 - 3x)e^{2x}$
D) $y = (5x - 3)e^{2x}$

100. The value of the integral $I = \int_0^1\int_0^1 e^{x+y}\, dx\, dy$ evaluated using Trapezoidal and Simpson's rule is I_T and I_S respectively. The value of $(I_T - I_S)$ is

A) 0.41219 B) 0.29545 C) 0.30762 D) 0.1217